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INSTITUTE FOR DEFENSE ANALYSES
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ABSTRACT

This paper documents a computer program to be used in solving nonlinear programming problems with nonlinear programming problems in the constraints. The program, named INSUMT, is used with the standard program, named SUMT, which implements the sequential unconstrained minimization technique for nonlinear programming. SUMT calls INSUMT when it is necessary to solve a nonlinear program in a constraint. The INSUMT program, together with a fairly complete example of its use, is included in the documentation.

Theory and applications of the models which can be solved using this program are treated in two companion papers.

CONTENTS

INTRODUCTION

Reference [2] formulates a class of mathematical programs with optimization problems in the constraints. Reference [3] gives models of a number of defense problems which can be approached using the techniques. The present paper documents a computer program for solving mathematical programs with nonlinear programming problems in the constraints.

Fiacco and McCormick in Reference [4] present theory and computational aspects of the sequential unconstrained minimization technique (SUMT) for nonlinear programming. Applications of nonlinear programming to a number of practical problems are presented in Reference [1]. The initial computer program for SUMT is documented in Reference [5]. The most recent computer program for SUMT is documented in Reference [6]. It handles equality constraints, and includes routines for computation of first and second partial derivatives using function values. Various unconstrained minimization methods are available in the SUMT program, requiring function values and first partial derivatives or requiring function values and first and second partial derivatives. Numerical differentiation routines facilitate the use of various unconstrained minimization methods. These are necessary in solving mathematical programs with nonlinear programs in the constraints, since the partial derivatives of the constraints cannot be stated explicitly.

The present paper is designed to be a supplement to Reference [6] and to be used with it.

PROBLEM DESCRIPTION AND SUMMARY OF COMPUTATIONAL PROCEDURE

One of the mathematical programs considered in Reference [2] is to choose vectors $x = (x_1, \ldots, x_n)$ and $v^i = (v_1^i, \ldots, v_{k_i}^i)$ for $i = 1, \ldots, m$ to

$$\underset{x \, \varepsilon \, X}{\text{minimize}} \; f(x)$$

subject to

$$h_i(x) = \min_{v^i \, \varepsilon \, V^i} g^i(x, v^i) \geq 0 \quad , \quad i = 1, 2, \ldots, m \quad .$$

It is shown there that if $g^i(x, v^i)$ is concave in x on a convex set X for every $v^i \, \varepsilon \, V^i$, then $h_i(x)$ is concave on X where the convex set X may be defined by inequality and/or equality constraints. If, in addition, $f(x)$ is a convex function on X, then the mathematical program is convex.

To outline the computational technique, it is useful to differentiate between the "outside program,"

$$\min_{x \, \varepsilon \, X} f(x)$$

subject to

$$h_i(x) \geq 0 \quad , \quad i = 1, 2, \ldots, m \quad ,$$

and the "ith inside program,"

$$\min_{v^i \, \varepsilon \, V^i} g^i(x, v^i) \quad .$$

When convenient the distinction among the m inside programs will be dropped, considering only the generic problem:

$$\min_{v \in V} g(x, v) \quad .$$

The constraint functions $h_i(x)$ in the outside problem are implicit in that their values depend upon the solution of the inside problem, which in turn depends upon the value of x. Thus, a solution technique for the overall problem must not rely on an explicit functional form for $h_i(x)$.

The computer program described in this paper, called INSUMT, is based on SUMT. The standard SUMT program is used for the outside problem. The new INSUMT program is used to solve the inside problem. SUMT and INSUMT are iterative routines. Let x^k denote the value of x for iteration k of the outside problem and v^ℓ denote iteration ℓ for the inside problem. The solution procedure is initialized by the user supplying x^o and v^o.

During iteration k for the outside problem, the inside problem is to choose $v \in V$ to minimize $g(x^k, v)$. At iteration ℓ of the inside problem, there is a value v^ℓ. This value is then used by INSUMT to generate a new value $v^{\ell+1}$. Continuing in this manner, the inside problem is solved, yielding $v^*(x^k)$ such that

$$g(x^k, v^*(x^k)) = \min_{v \in V} g(x^k, v) \quad .$$

Control then passes to the outside problem which generates a new solution value x^{k+1}. The process is repeated until the sequence $\{x^k\}$ converges.

The routine INSUMT resides in core with SUMT and may be called to evaluate more than one constraint of the outside problem. For each such constraint, user-supplied subroutines of INSUMT provide information concerning the functional form of $g^i(x,v^i)$ and of the constraint set V^i. User-supplied subroutines for SUMT give analogous information for $f(x)$ and X.

The SUMT and INSUMT programs iteratively generate values of the solution variable by optimizing an unconstrained penalty function. Specifically, iteration $\ell + 1$ generates $v^{\ell+1}$ from v^{ℓ} and from the functional forms of the objective function of the inside program and the constraint set. The computation calls for the first, and sometimes second, partial derivatives of the relevant functions. These may be specified by the user. However, for the outside problem, explicit derivatives of $h_i(x)$ cannot be given. SUMT uses numerical differentiation in this case to approximate the derivatives to be used in generating x^{k+1} from x^k.

DESCRIPTION OF COMPUTER PROGRAM

The program is modular in structure. It consists basically of the SUMT program (subroutines MAIN through XMOVE), one set of user-supplied subroutines containing function evaluations for the outside mathematical program (READIN, RESTNT, GRAD1 and MATRIX), the INSUMT program (subroutines SUB through XMOVES), and one set of user-supplied subroutines containing function evaluations for the one or more inside mathematical programs (READIX, RESTNX, GRAD1S and MATRXX).

The workings of the SUMT subroutines MAIN through XMOVE are described in Reference [6]. The user-supplied subroutines for the outside program perform the following tasks when used with INSUMT.

READIN reads in the parameters used in evaluating the objective function and constraints.

RESTNT evaluates the objective function and constraints of the outside program for a value of x . Except where constraints contain inside mathematical programs, the functions are evaluated for the current value of x . Otherwise, RESTNT calls SUB, identifying the inside mathematical program to be solved for the current value of x , and SUB returns the value $v^*(x)$. RESTNT then evaluates $h(x) = g(x, v^*(x))$.

GRAD1 evaluates the first partial derivatives of the objective function and constraints in two ways. Where the constraints do not contain inside mathematical programs, for the current x the first partial derivatives are evaluated either explicitly or using numerical differentiation. Otherwise, GRAD1 uses DIFF1 to numerically differentiate the constraints which include inside mathematical programs.

MATRIX evaluates the second partial derivatives of the objective function and constraints of the outside mathematical program in two

ways. Where the constraints do not contain inside mathematical programs, for the current x the second partial derivatives are evaluated either explicitly or using numerical differentiation. Otherwise, MATRIX uses DIFF2 to numerically differentiate the constraints which include inside mathematical programs.

The INSUMT program consists of subroutines SUB through XMOVES. Three basic changes are made in SUMT to result in INSUMT.

First, all subroutine names are changed by simply adding an S to the end except where this results in more than six letters, in which case the last two letters are replaced by X (e.g., OPT becomes OPTS and RESTNT becomes RESTNX). All calls to subroutines within INSUMT are changed to include the revised names of the called subroutines. SUMT calls INSUMT only by RESTNT calling SUB. After SUB is called INSUMT calls only the subroutines within INSUMT until SUB returns to RESTNT.

Second, all labeled COMMON arrays are changed by adding an S to the end, except where this results in more than six letters, in which case the last two letters are replaced by X (e.g., SHARE becomes SHARES and CONPAR becomes CONPAX).

Third, subroutine MAIN is modified to obtain subroutine SUB by changing the name from MAIN to SUB, and by modifying the subroutine to read in data for the inside mathematical programs the first time each problem is solved and to save the data in an array for subsequent solutions of the problems.

User-supplied subroutines READIX, RESTNX, GRAD1S and MATRXX are called by INSUMT. Depending on a parameter denoting the inside mathematical program being solved, the subroutines calculate appropriate function values, first partial derivatives, or second partial derivatives.

In the first several runs of a program, it is necessary to observe the intermediate points of the solution of the inside programs. RESTNT thus prints out which inside program is being solved, and INSUMT prints out the intermediate points. After it

is established that the inside programs are being successfully solved, printing of their points is suppressed by deleting the print statements from RESTNT and by modifying BODYS, CONVRX, ESTIMS, FEASS, INVERX, OPTS, OUTPUX, PUNCHS, TECHECX and TIMECS. The modifications are described later.

EXAMPLE PROBLEM

An example problem is used to describe the use of INSUMT with SUMT.

The problem is to choose $x_1, \ldots, x_n$ to

$$\text{minimize } x_1 + \ldots + x_n$$

subject to

$$\left[\min_{v_1 + \ldots + v_n \leq n} x_1(v_1 - 2)^2 + \ldots + x_n(v_n - 2)^2\right] \geq r_1$$

$$\left[\min_{v_1^2 + \ldots + v_n^2 \leq n} x_1^{.5}(v_1 - 2)^2 + \ldots + x_n^{.5}(v_n - 2)^2\right] \geq r_2 .$$

To illustrate the problem, take $n = 4$, $r_1 = 4$, $r_2 = 4$ and let the starting point be $x_1 = x_2 = x_3 = x_4 = 4$ for the outside program and $v_1 = v_2 = v_3 = v_4 = .5$ for both inside programs. The value of the objective function of the outside program is

$$x_1 + x_2 + x_3 + x_4 = 16 \quad .$$

The objective function of the first inside program is

$$4(.5 - 2)^2 + 4(.5 - 2)^2 + 4(.5 - 2)^2 + 4(.5 - 2)^2 = 36 \quad ,$$

and since $.5 + .5 + .5 + .5 = 2 \leq 4$ the first inside program starting point is feasible. For the second inside program the objective

function is

$$2(.5 - 2)^2 + 2(.5 - 2)^2 + 2(.5 - 2)^2 + 2(.5 - 2)^2 = 18 \quad ,$$

and since $.5^2 + .5^2 + .5^2 + .5^2 = 1 \leq 4$ the second inside program starting point is feasible. Since $36 > r_1 (= 4)$, $18 > r_2 (= 4)$, the three starting points, one outside and two inside, provide a feasible point for all three programs.

The solution to the example is $x_1 = x_2 = x_3 = x_4 = 1$, with $v_1 = v_2 = v_3 = v_4 = 1$ in both inside programs, yielding

$$x_1 + x_2 + x_3 + x_4 = 1 + 1 + 1 + 1 = 4$$

and

$$\begin{bmatrix} 1(1 - 2)^2 + 1(1 - 2)^2 + 1(1 - 2)^2 + 1(1 - 2)^2 = 4 \\ \text{s.t.} \\ (1 + 1 + 1 + 1 = 4) \leq 4 \end{bmatrix} \geq 4$$

$$\begin{bmatrix} 1^2(1 - 2)^2 + 1^2(1 - 2)^2 + 1^2(1 - 2)^2 + 1^2(1 - 2)^2 = 4 \\ \text{s.t.} \\ (1^2 + 1^2 + 1^2 + 1^2 = 4) \leq 4 \end{bmatrix} \geq 4 \quad .$$

COMPUTER PROGRAM INCLUDING USER-SUPPLIED SUBROUTINES FOR EXAMPLE PROBLEM

In this section the user-supplied subroutines for the outside program are presented, followed by the INSUMT program, followed by the user-supplied subroutines for the inside programs. The SUMT program is not supplied, being documented in Reference [6].

RESTNT contains print statements for each call for solution and end of solution of an inside program. The INSUMT subroutines SUB, BODYS, CHCKEX, CONVRX, ESTIMS, FEASS, INVERX, OPTS, OUTPUX, PUNCHS, TCHECX and TIMECS contain print statements for the points of the inside programs. Thus the program as listed prints all intermediate points. However, the changes necessary to suppress printing are indicated in heavy markings directly on the listing. Simply remove the boxed statements, and replace them by the statements written, if any. Printing is done by SUB only the first time it is called, so SUB is not modified. Printing is done by CHCKEX only if control cards dictate, so it is not modified. All other print statements are removed to suppress printing of inside programs.

It should be noted that all data of the example problem are contained in the set of user-supplied subroutines for the outside program (READIN, RESTNT, GRAD1, and MATRIX) and the set of user-supplied subroutines for the inside programs (READIX, RESTNX, GRAD1S and MATRXX). For most problems READIN should be used to read data for outside and inside programs, since READIX is called each time an inside program is solved, and it should not be used.

Control cards are read in the following order: Outside program, first inside program, second inside program, and so on if there are

more inside programs. SUB saves the control card data, and inside program starting points, and provides these data each time an inside program is solved. Dimensions on PARS are presently (2,47), including up to 20 variables in the inside program. This would have to be changed for more than 2 inside programs or 20 variables.

User-Supplied Subroutines for Outside Program

```
                  SUBROUTINE READIN
000002      9999  CONTINUE
000002            RETURN
000003            END
```

```
              SUBROUTINE RESTNT(IN,VAL)
000005        COMMON/PROB/ISP
000005        COMMON/OUT/NOUT,XT(100),V(100)
000005        COMMON/SHARE/ X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1          10000070
000005        FN=N
000006        VAL=0.
000007        IF(IN)100,100,200
000010    100 DO 150 J=1,N
000012    150 VAL = VAL + X(J)
000016        GO TO 9999
000016    200 IF(IN-1)300,300,400
000020    300 ISP=1
000021        NOUT=N
000023        DO 305 J=1,N
000024        IF(X(J)-0.)304,304,305
000027    304 X(J)=1.E-08
000031    305 XT(J)=X(J)
000036        WRITE(6,307)
000041    307 FORMAT(11H1 PROBLEM A)
000041        CALL SUB
000042        WRITE(6,308)
000046    308 FORMAT(18H1 PROBLEM A SOLVED)
000046        DO 320 J=1,N
000051    320 VAL = VAL + X(J)*(V(J)-2.)**2
000057        VAL=VAL-4.
000061        GO TO 9999
000061    400 ISP=2
000062        NOUT=N
000064        DO 405 J=1,N
000065        IF(X(J)-0.)404,404,405
000070    404 X(J)=1.E-08
000072    405 XT(J)=X(J)
000076        WRITE(6,407)
000102    407 FORMAT(11H1 PROBLEM B)
000102        CALL SUB
000103        WRITE(6,408)
000107    408 FORMAT(18H1 PROBLEM B SOLVED)
000107        DO 420 J=1,N
000112    420 VAL = VAL + X(J)**.5 * (V(J)-2.)**2
000124        VAL=VAL-4.
000126        GO TO 9999
000126   9999 CONTINUE
000126        RETURN
000127        END
```

```
                 SUBROUTINE GRAD1(IN)
000003           COMMON/SHARE/ X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1        10000070
000003           DO 50 J=1,N
000005        50 DEL(J)=0.
000010           IF(IN) 100,100,200
000011       100 DO 150 J=1,N
000013       150 DEL(J) = 1.
000017           GO TO 9999
000017       200 CALL DIFF1(IN)
000020           GO TO 9999
000022      9999 RETURN
000023           END
```

```
      SUBROUTINE MATRIX(IN,L)
      COMMON/SHARE/ X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1         10000070
      IF(IN)100,100,200
  100 L=1
      GO TO 9999
  200 CALL DIFF2(IN)
      GO TO 9999
 9999 CONTINUE
      RETURN
      END
```

INSUMT

```
             SUBROUTINE SUB
        C
        C               MARCH 1971
        C
        C MAIN IS THE PROGRAM THAT INITIATES THE SUMT ALGORITHM. THE INPUT OF
        C PARAMETERS, OPTIONS, AND STARTING POINT IS DONE IN MAIN. AFTER THE
        C SOLUTION OF ONE NLP PROBLEM MAIN LOOKS FOR DATA FOR ANOTHER NLP PROB.
        C
000002       COMMON/PROB/ISP
000002       COMMON/OUT/NOUT,XT(100),V(100)
000002       COMMON/IN/W(100)
        C
000002       COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NPI,NMI
000002       COMMON /EQALS/H, HI, MZ
000002       COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002       COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RH0
000002       COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,
            1 RSIG1, GI,     XI(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
            2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
            3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000002       COMMON/EXPOPX / NEXOP1, NEXOP2, XEP1, XEP2
        C
000002       DIMENSION IFTS(2),PARS(2,47)
000002       DATA IFTS/0,0/
        C
000002       DO 5 J=1,NOUT
000004     5 W(J)=XT(J)
000010       IF(IFTS(ISP))10,10,20
        C
        C    PARAMETER CARD
000012  10   READ (5,50) EPSI,RHOIN,THETA0,RATIO,TMMAX,M,N,MZ
        C    INITIAL  X VERCTOR CARD  FORMAT
000036       READ (5,60) (X(I),I=1,N)
000051       NTCTR=0
000052       NP1=N+1
000054       NM1=N-1
        C SUBROUTINE READIN IS UNDER PROGRAMMER CONTROL
000055       CALL READIX
        C OPTION CARD FOLLOWS PROGRAMMERS DATA
000056       READ (5,80) NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000106       WRITE (6,110)
000112       WRITE (6,120) N,M,MZ,TMMAX,RHOIN,RATIO,EPSI,THETA0
000136       WRITE (6,130)
000142       WRITE (6,80) NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
        C--READ TOLERANCES
000172       READ (5,60) XEP1, XEP2
000202       WRITE (6,90)
000206       WRITE (6,70) XEP1, XEP2
        C--READ SECOND OPTION CARD
000216       READ (5,80) NEXOP1,NEXOP2
000226       WRITE (6,100)
000232       WRITE (6,80) NEXOP1,NEXOP2
000242       PARS (ISP, 1) = EPS1
000244       PARS(ISP, 2) = RHOIN
000246       PARS (ISP, 3) = THETA0
000247       PARS (ISP, 4) = RATIO
000251       PARS (ISP, 5) = TMMAX
```

```
000252          PARS (ISP, 6) = M
000254          PARS (ISP, 7) = N
000256          PARS (ISP, 8) = MZ
000260          DO 4 K=1,20
000262          L=K+8
000264        4 PARS(ISP,L) = X(K)
000271          PARS (ISP,29) = NT1
000273          PARS (ISP,30) = NT2
000275          PARS (ISP,31) = NT3
000277          PARS (ISP,32) = NT4
000301          PARS (ISP,33) = NT5
000303          PARS (ISP,34) = NT6
000305          PARS (ISP,35) = NT7
000307          PARS (ISP,36) = NT8
000311          PARS (ISP,37) = NT9
000313          PARS (ISP,38) = NT10
000315          PARS ( ISP,39) = NT11
000317          PARS (ISP,40) = XEP1
000320          PARS (ISP,41) = XEP2
000322          PARS (ISP,42) = XEP3
000323          PARS (ISP,43) = NEXOP1
000325          PARS (ISP,44) = NEXOP2
000327          PARS (ISP,45) = NEXOP3
000331          PARS (ISP,46) = NEXOP4
000333          PARS (ISP,47) = NEXOP5
000335          IFTS(ISP)=1
000337          GO TO 25
      C
000337       20 EPSI = PARS(ISP,1)
000341          RHOIN= PARS(ISP,2)
000343          THETA0 = PARS(ISP,3)
000344          RATIO  = PARS(ISP,4)
000346          TMMAX  = PARS(ISP,5)
000347          M = PARS(ISP,6)
000351          N = PARS(ISP,7)
000353          MZ= PARS(ISP,8)
000355          DO 21 K=1,20
000357          L= K+8
000361       21  X(K) = PARS(ISP,L)
000366          NT1  = PARS(ISP,29)
000370          NT2  = PARS(ISP,30)
000372          NT3  = PARS(ISP,31)
000374          NT4  = PARS(ISP,32)
000376          NT5  = PARS(ISP,33)
000400          NT6  = PARS(ISP,34)
000402          NT7  = PARS(ISP,35)
000404          NT8  = PARS(ISP,36)
000406          NT9  = PARS(ISP,37)
000410          NT10 = PARS(ISP,38)
000412          NT11 = PARS(ISP,39)
000414          XEP1 = PARS(ISP,40)
000415          XEP2 = PARS(ISP,41)
000417          XEP3 = PARS(ISP,42)
000420          NEXOP1=PARS(ISP,43)
000422          NEXOP2=PARS(ISP,44)
000424          NEXOP3=PARS(ISP,45)
000426          NEXOP4=PARS(ISP,46)
000430          NEXOP5=PARS(ISP,47)
```

```
000432          GO TO 25
           C
000433       25 CALL SETS(TMMAX)
000435          CALL TIMECS
000436          NPHASE=4
           C --- JUST TO GET AN INITIAL PRINTOUT
000437          CALL EVALUS
000440          P0=0.0
000441          G=0.0
000442          H=0.0
000443          RSIGMA=0.0
000444          CALL OUTPUX (2)
000445          CALL STORES
000446          IF (NEXOP1.GT.1) CALL CHCKEX
000452          IF (NEXOP1.EQ.3) STOP 01072
000456          IF (NEXOP1.EQ.5) STOP 01104
000462          CALL FEASS
           C NPHASE=5 IS USED TO INDICATE NO FEASIBLE POINT EXIST
000463          GO TO (30,30,30,30,10), NPHASE
000474     30   NPHASE=2
000475          NTCTR=0
000476          CALL BODYS
000477          DO 35 J=1,N
000501       35 V(J)=X(J)
000505          RETURN
           C
           C    PARAMETER CARD
000505     50   FORMAT (5E12.0,3I4)
           C    INITIAL  X VERCTOR CARD  FORMAT
000505     60   FORMAT (6E12.6)
000505     70   FORMAT (6E20.7)
           C    OPTION CARD FORMAT
000505     80   FORMAT (10I7)
000505     90   FORMAT(13H0 TOLERANCES  )
000505     100  FORMAT (26H0 SECOND SET OF OPTIONS    )
000505     110  FORMAT (56H1 NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4    SUB
               I    )
000505     120  FORMAT (1H0,5X,2HN=I3,6X,2HM=I3,6X,3HMZ=I3//8X,10HMAX. TIME=E14.7,
               I4X,2HR=EI4.7,4X,6HRATIO=EI4.7,6X,8HEPSILON=E14.7,4X,6HTHETA=E14.7)
000505     130  FORMAT (18H0 OPTIONS SELECTED)
000505          END
```

```
              SUBROUTINE BODYS
          C
          C             OCTOBER 1970
          C
          C BODY COORDINATES THE FLOW AMONG THE SUBROUTINES THAT ACTUALLY DO THE
          C CALCULATIONS REQUIRED BY THE VARIOUS PARTS OF THE ALGORITHM.
000002        COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000002        COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002        COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000002        COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,
             1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
             2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
             3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000002        COMMON/CONPAX / NF1, NF2,NF3
000002        NF2=2
000003        NF3=2
000004        MN=0
000005        NUMINI=0
          C     OPTION OF GETTING INITIAL RHO
000006        CALL RHOCOX
000007        CALL EVALUS
000010    10  CALL XMOVES
000011        GO TO (30,20), NT3
000017    20  CALL TIMECS
000020        CALL OUTPUX (1)
000022        GO TO 40
000023     30 CALL TCHECX
          C IN FEASIBILITY PHASE 4 MEANS  FEAS ACHIEVED
000024    40  GO TO (50,50,50,200), NSATIS
000034     50 CALL CONVRX (N1)
000036        GO TO (60,10,130), N1
          C  MINIMUM ACHIEVED IF  N1=1
000045    60  GO TO (70,80), NT3
000053    70  CALL TIMECS
000054        CALL OUTPUX (1)
          C --- NUMBER OF MINIMA ACHIEVED INCREASED BY 1
000056    80  NUMINI=NUMINI+1
000060        MN=0
000061        GO TO (190,90,90), NPHASE
000070    90  CALL ESTIMS
          C  FINAL RIGHT HAVE BEEN CALLED BY ESTIM_CONVERGED IF N2=1
000071        GO TO (100,110,120), NT4
          C   NT4=1 FINAL CONVERGENCE ON 0 ORDER ESTIMATES, NT4=2 CONVERGE ON FIRS
          C  ORDER ESTIMATES, NT4=3 CONVERGE ON SECOND ORDER ESTIMATES.
000100    100 CALL FINALS(NF1)
000102        GO TO (130,140), NF1
000110    110 GO TO (130,140), NF2
000116    120 GO TO (130,140), NF3
000124    130 RETURN
000125    140 RHO=RHO/RATIO
          C USING PREVIOSLY COMPUTED VALUES FOR F, AND RJ P IS RECOMPUTED WITH THE
          C NEW VALUE OF RHO.
000127        CALL PEVALX
          C A VECTOR IS LEFT IN DELX(I) BY ESTIM
000130        IF (NUMINI-2) 10,150,150
000133    150 GO TO (10,160,160), NT7
000142    160 CALL GRADS(2)
```

```
000144          CALL OPTS
000145          GO TO (180,170), NT3
000153   170    WRITE (6,210)                    170 CONTINUE
000157          CALL OUTPUX (1)
000161   180    GO TO 50
000162   190    IF (G) 90,90,200
000164   200    RETURN
         C ---  DUAL VALUE GREATER THAN 0 MEANS NO FEASIBLE POINT EXISTS
         C
000165   210    FORMAT (6X,30HMOVED ON EXTRAPOLATION VECTOR )
000165          END
```

```
              SUBROUTINE CHCKEX
        C
        C             MARCH 1971
        C
        C CHCKER COMPUTES AND LIST THE FIRST PARTIAL DERIVATIVES USING GRAD1
        C AND THEN USING NUMERICAL DIFFERENCING (DIFF1).  IF REQUESTED THE
        C SECOND PARTIAL DERIVATIVES ARE COMPUTED AND LISTED USING MATRIX AND
        C DIFF2.
000002        COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000002        COMMON /EQALS/H, H1, MZ
000002        MMZ=1+M+MZ
000005        DO 5 J=1,N
000006        DEL(J)=1.2345678
000010  5     CONTINUE
000012        DO 10 I=1,MMZ
000013        IN=I-1
000015        WRITE (6,170) IN
000022        CALL GRADIS(IN)
000024        WRITE (6,180) (DEL(J),J=1,N)
000037        CALL DIFFIS(IN)
000041        WRITE (6,180) (DEL(J),J=1,N)
000054  10    CONTINUE
        C...SOMETIMES ONLY FIRST DERIVATIVES ARE TO BE CHECKED
000057        IF (NEXOP1.LT.4) GO TO 160
000061        WRITE (6,190)
000065        DO 150 I=1,MMZ
000067        IN=I-1
000071        WRITE (6,170) IN
000076        IT=2
000077        DO 30 K=1,N
000101        DO 20 J=1,N
000102  20    A(K,J)=0.
000110  30    CONTINUE
000112        CALL MATRXX(IN,IT)
000114        IF (IT.EQ.1) GO TO 150
000116        DO 50 K=2,N
000120        KM1=K-1
000122        DO 40 J=1,KM1
000123        IF (A(K,J).EQ.0.0) GO TO 40
000126        NEXOP1=5
000127        WRITE (6,210) K,J
000137        GO TO 60
000140  40    CONTINUE
000143  50    CONTINUE
000145  60    DO 90 K=1,N
000147        DO 70 J=K,N
000150        IF (A(K,J).NE.0.0) GO TO 80
000153  70    CONTINUE
000156        WRITE (6,220) K
000163        GO TO 90
000164  80    WRITE (6,200) K,(A(K,J),J=1,N)
000203  90    CONTINUE
000206        DO 110 K=1,N
000207        DO 100 J=1,N
000210  100   A(K,J)=0.
000216  110   CONTINUE
000220        WRITE(6,115) IN
```

```
000226    115   FORMAT (13H0 CALL DIFF2(,I2,1H) )
000226          CALL DIFF2S(IN)
000230          DO 140 K=1,N
000232          DO 120 J=K,N
000233          IF (A(K,J).NE.0) GO TO 130
000236    120   CONTINUE
000241          WRITE (6,220) K
000246          GO TO 140
000247    130   WRITE (6,200) K,(A(K,J),J=1,N)
000266    140   CONTINUE
000271    150   CONTINUE
000274    160   CONTINUE
000274          RETURN
          C
000275    170   FORMAT (   28H0CHECKER.....CONSTRAINT NO. , I3)
000275    180   FORMAT (1H0,24HCHECKER.....1ST PARTIALS/(1X,E20.8,E20.8,E20.8,E20.
               18,E20.8,E20.8))
000275    190   FORMAT (1H0,24HCHECKER.....2ND PARTIALS)
000275    200   FORMAT (4H0ROW, I3     /(1X,E20.8,E20.8,E20.8,E20.8,E20.8,E20.8))
000275    210   FORMAT (3H A(,I2,1H,,I2,10H) .NE. 0,0)
000275    220   FORMAT (4H ROW,I3,11H ALL ZEROS.)
000275          END
```

```
              SUBROUTINE CONVRX (N1)
          C
          C               OCTOBER 1970
          C
          C AFTER EACH ITERATION OF THE ALGORITHM TO LOCATE THE MINIMUM OF THE
          C PENALTY FUNCTION, CONVRG DETERMINES IF THE CURRENT POINT IS CLOSE
          C ENOUGH TO THE POINT GIVING THE MINIMUM VALUE OF THE P FUNCTION.
          C       N1 SET EQUAL TO 1 IF MINIMUM HAS BEEN FOUND.
          C       N1 SET EQUAL TO 2 IF MINIMUM HAS NOT BEEN FOUND AND TIME IS NOT UP
          C       N1 SET EQUAL TO 3 OTHERWISE
          C       DOTT SET EQUAL TO (DEL P)(INVERSE(DEL(DEL P)))(DEL P) IN OPT
000003            COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000003            COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000003            COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000003            COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,
                 1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
                 2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
                 3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000003            COMMON/EXPONX / NEXOP1, NEXOP2, XEP1, XEP2
000003            COMMON /TSWS/NSWW
000003            N1=2
000004            IF (MN.LE.1) Q1=P0
000011            GO TO (10,20,30), NT9
000020    10      IF (ABS(DOTT).LT.EPSI) GO TO 70
000024            GO TO 40
000024    20      IF (ABS(DOTT).LT.(P1-P0)/5.0) GO TO 70
000032            GO TO 40
000032    30      IF (ADELX.LT.EPSI) GO TO 70
000035    40      GO TO (50,60), NSWW
000043    50      IF (MN.LE.1) RETURN
000047            IF (P0.XEP2 .LT. Q1) GO TO 75
000053            WRITE (6,80)
000056            GO TO 70
000060    60      CALL PUNCHS
000061            WRITE (6,90)
000065            N1=3
          C
          C       FOUND THE MINIMUM TO THE SUBPROBLEM.
000067            RETURN
000070    70      N1=1
000071    75      Q1 = P0
000073            RETURN
          C
000073    80      FORMAT (100H APPARENTLY ROUNDOFF ERRORS PREVENT A MORE ACCURATE DE
                 1TERMINATION OF THE MINIMUM OF THIS SUBPROBLEM.)
000073    90      FORMAT (48H0**** TIME IS UP, CALLING EXIT FROM CONVRG. ****)
000073            END
```

```
              SUBROUTINE DIFFIS(IN)
       C
       C              FEBUARY 1971
       C
       C DIFF1 COMPUTES THE FIRST DERIVATIVES BY NUMERICAL DIFFERENCING.
       C
       C--USER CAN CALL FOR DIFFERENCING OF SELECTED FUNCTIONS
000003        COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000003        COMMON/EXPOPX / NEXOP1, NEXOP2, XEP1, XEP2
000003        COMMON/ STIRXS/XSTR( 100 )  ,  XSSS( 100), DDLL(100)
000003        DO 10 J=1,N
000005 10     XSTR(J)=X(J)
000011        DO 30 J=1,N
000012        IF (J.EQ.1) GO TO 20
000014        JM1=J-1
000015        X(JM1)=XSTR(JM1)
000017 20     X(J)=XSTR(J)+XEP1
000022        CALL RESTNX (IN,ZZ2)
000024        X(J)=XSTR(J)-XEP1
000027        CALL RESTNX (IN,ZZ1)
000032 30     DEL(J)=(ZZ2-ZZ1)/(2.*XEP1)
000042        X(N)=XSTR(N)
000043        RETURN
000044        END
```

```
              SUBROUTINE DIFF2S(IN)
         C
         C             OCTOBER 1970
         C
         C DIFF2 COMPUTES THE SECOND DERIVATIVES BY NUMERICAL DIFFERENCING.
         C
000003        COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000003        COMMON/EXPOPX / NEXOP1, NEXOP2, XEP1, XEP2
000003        COMMON/ STIRXS/XSTR( 100 )  ,  XSSS( 100), DDLL(100)
000003        DO 10 J=1,N
000005   10   XSSS(J)=X(J)
000011        DO 50 J=1,N
000012        IF (J.EQ.1) GO TO 20
000014        JM1=J-1
000015        X(JM1)=XSSS(JM1)
000017   20   X(J)=XSSS(J)+XEP1
000022        CALL GRAD1S(IN)
000023        DO 30 I=1,N
000026   30   DDLL(I)=DEL(I)
000032        X(J)=XSSS(J)-XEP1
000035        CALL GRAD1S(IN)
000036        DO 40 I=J,N
000041   40   A(J,I)=(DDLL(I)-DEL(I))/(2.*XEP1)
000057   50   CONTINUE
000061        X(N)=XSSS(N)
000062        RETURN
000063        END
```

```
              SUBROUTINE ESTIMS
          C                                                                  003310
          C             OCTOBER 1970                                         003320
          C                                                                  003330
          C ESTIM PERFORMS THE COMPUTATIONS TO ESTIMATE THE LAGRANGE MULTIPLIERS  003340
          C AND MAKE THE FIRST- AND SECOND-ORDER ESTIMATES OF THE FINAL SOLUTION  003350
          C OF THE PROBLEM.                                                  003360
000002        COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000002        COMMON /EQALS/H, H1, MZ
000002        COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002        COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000002        COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,
             1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
             2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
             3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000002        COMMON/CONPAX / NF1, NF2,NF3
000002        CALL STORES
000003        Z10=RATIO**2                                                   003470
000005        Z9=RATIO                                                       003480
000006        Z1=1.0/Z9+1.0/Z10                                              003490
000011        Z2=Z1+1./Z9**3                                                 003500
000014        Z3=1./Z9**3                                                    003510
000016        Z4=Z10+Z9                                                      003520
000017        Z5=Z9**3                                                       003530
000020        Z6=1.0/((Z10-1.0)*(Z9-1.0))                                    003540
000024        Z7=1./Z9                                                       003550
000026        Z8=1./(Z9-1.)                                                  003560
000030        RQ=1.0/RHO                                                     003570
000032        IF (NUMINI-2) 150,80,10                                        003580
000034   10   WRITE (6,330)                        10 CONTINUE
000040        P0=(PR2-Z4*PR1+Z5*P1)*Z6                                       003600
000047        G=(RATIO*G1-GR1)/(RATIO-1.)                                    003610
000053        DO 20 I=1,N                                                    003620
000055   20   X(I)=(XR2(I)-Z4*XR1(I)+Z5*X1(I))*Z6                            003630
000066        NP=NPHASE                                                      003640
000070        NPHASE=4                                                       003650
000071        CALL EVALUS
000072        NPHASE=NP                                                      003670
000074        CALL OUTPUX (2)
          C CHECK TO SEE IF ESTIMATES HAVE CONVERGED                         003690
000075        GO TO (70,30,70), NPHASE                                       003700
000104   30   DO 50 J=1,M                                                    003710
000106        IF (RJ(J)) 40,50,50                                            003720
000110   40   IF (THETA0+RJ(J)) 70,50,50                                     003730
000113   50   CONTINUE                                                       003740
000116        GO TO (70,70,60), NT4                                          003750
000125   60   CALL FINALS(NF3)
000127   70   CONTINUE                                                       003770
000127   80   WRITE (6,340)                        80 CONTINUE
000133        G=(RATIO*G1-GR1)/(RATIO-1.)                                    003800
000140        P0=(Z9*P1-PR1)*Z8                                              003790
000144        DO 90 I=1,N                                                    003810
000145   90   X(I)=(Z9*X1(I)-XR1(I))*Z8                                      003820
000153        NP=NPHASE                                                      003830
000155        NPHASE=4                                                       003840
000156        CALL EVALUS
000157        NPHASE=NP                                                      003860
```

```
000161          CALL OUTPUX (2)
        C CHECK TO SEE IF ESTIMATES HAVE CONVERGED
000162          GO TO (140,100,140), NPHASE
000171   100    DO 120 J=1,M
000173          IF (RJ(J)) 110,120,120
000175   110    IF (RJ(J)+THETA0) 140,120,120
000200   120    CONTINUE
000203          GO TO (140,130,140), NT4
000212   130    CALL FINALS(NF2)
000214   140    CONTINUE
000214   150    WRITE (6,350)                      150 CONTINUE
000220          IF (M) 180,180,160
000222   160    DO 170 J=1,M
000224   170    RJ(J)=RHO/RJ1(J)
000230   180    IF (MZ) 210,210,190
000232   190    DO 200 J=1,MZ
000234          MNJ=M+J
000236   200    RJ(MNJ)=2.*RJ1(MNJ)*RO
000244   210    GO TO (220,240), NT2
000252   220    DO 230 I=1,N
000254   230    X(I)=RHO/X1(I)
000260   240    CALL OUTPUX (2)
000262          CALL REJECX
000263          IF (NUMINI-2) 280,300,250
000266   250    GO TO (280,310,260), NT7
        C       SECOND ORDER MOVE FOR NEXT MINIMUM
000275   260    DO 270 I=1,N
000277   270    DELX(I)=Z1*X1(I)-Z2*XR1(I)+Z3*XR2(I)
000310   280    PR2=PR1
000312          GR2=GR1
000313          PR1=P1
000314          GR1=G1
000315          DO 290 I=1,N
000317          XR2(I)=XR1(I)
000321   290    XR1(I)=X1(I)
000324          RETURN
000325   300    GO TO (280,310,310), NT7
000334   310    DO 320 I=1,N
000336   320    DELX(I)=(X1(I)-XR1(I))*Z7
000343          GO TO 280
        C
000344   330    FORMAT (/26H0    2ND  ORDER ESTIMATES )
000344   340    FORMAT (/26H0    1ST  ORDER ESTIMATES )
000344   350    FORMAT (/25H0  LAGRANGE MULTIPLIERS  )
000344          END
```

```
              SUBROUTINE EVALUS
          C
          C                OCTOBER 1970
          C
          C IN THE NORMAL PHASE EVALU CALLS THE USER-SUPPLIEO ROUTINES TO EVALUATE
          C THE OBJECTIVE FUNCTION ANO THE CONSTRAINT FUNCTIONS AT THE CURRENT
          C POINT. IN THE FEASIBILITY PHASE THIS ROUTINE PUTS THE NEGATIVE SUM OF
          C THE VIOCATED CONSTRAINTS IN LOCATION F.
000002        COMMON/SHARES/X(100), OEL(100), A(100,100),N,M, MN,NP1,NM1
000002        COMMON /EQALS/H, H1, MZ
000002        COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002        COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000002        COMMON/CRSTS/DELX(100), OELX0(100), RHOIN,RATIO, EPSI, THETA0,
             1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
             2 PR2,P1, F1, RJ1(200), 0OTT, PGRA0(100), OIAG(100),
             3 PREV3'AOELX' NTCTR' NUMINI' NPHASE' NSATIS
000002        H=0.0
000003        RSIGMA=0.0
000004        F=0.0
000005        NSATIS=2
000006        GO TO (10,100,190,200), NPHASE
          C       =1  FEASIBILITY
          C       =2  NORMAL
          C       =3  GUESS
          C       =4  ALL FUNCTIONS ARE TO BE EVALUATEO
          C FEASIBILITY
000016    10  GO TO (20,40), NT2
          C NON-NEGATIVIES INCLUOED
000024    20  O0 30 I=1,N
000026        IF (X(I)) 260'260'30
000030    30  RSIGMA=RSIGMA-RHO*ALOG(X(I))
000040    40  IF (M.EQ.0) GO TO 90
000041        O0 80 J=1,M
000043        CALL RESTNX (J,RJ(J))
000045        IF (RJ1(J).LE.0.0) GO TO 50
000047        IF (RJ(J).GT.0.0) GO TO 60
          C VIOLATION OF A PREVIOUSLY SATISFIED CONSTRAINT
000052        GO TO 260
000052    50  IF (RJ(J).GT.0.0) GO TO 70
          C ALL VIOLATEO CONSTRAINTS ADDEO INTO OBJECTIVE FUNCTION
000055        F=F-RJ(J)
000057        GO TO 80
000057    60  RSIGMA=RSIGMA-RHO*ALOG(RJ(J))
000064        GO TO 80
          C INOICATES SATISFACTION OF CONSTRAINT(1ORMORE)
000065    70  NSATIS=1
000066        RSIGMA=RSIGMA-RHO*ALOG(RJ(J))
000073    80  CONTINUE
000076    90  CONTINUE
          C EQUALITIES NOT COMPUTEO IN FEAS. PHASE
000076        P0=F+RSIGMA
000100        G=F-RHO*FLOAT(M)
000105        IF (NT2.EQ.1) G=G-RHO*FLOAT(N)
000112        RETURN
          C  REGULAR  PHASE
000113    100 GO TO (110'130)' NT2
          C NON NEGATIVITIES INCLUOEO
```

```
000121   110   DO 120 I=1,N
000123         IF (X(I)) 260,260,120
000125   120   RSIGMA=RSIGMA-RHO*ALOG(X(I))
000135   130   IF (M.EQ.0) GO TO 150
000136         DO 140 J=1,M
000140         CALL RESTNX (J,RJ(J))
000142         IF (RJ(J).LE.0.0) GO TO 260
000144         RSIGMA=RSIGMA-RHO*ALOG(RJ(J))
000151   140   CONTINUE
         C EVALUATE AND ADD IN EQUALITY CONSTRAINTS
000154   150   CONTINUE
000154         CALL RESTNX ( 0,F)
000156         IF (MZ) 180,180,160
000160   160   DO 170 I=1,MZ
000162         J=I+M
000164         CALL RESTNX (J,RJ(J))
         C ADD INTO THIRD TERM OF P FUNCTION
000166         H=H+(RJ(J))**2
000171   170   CONTINUE
000174         H=H/RHO
000175   180   P0=RSIGMA+H
000177         P0=F+P0
000201         G=2.*H-RHO*FLOAT(M)
000205         G=G+F
000207         IF (NT2.EQ.1) G=G-RHO*FLOAT(N)
         C DUAL VALUE
000214         RETURN
         C GUESS PHASE NOT CODED
000215   190   RETURN
         C---  STRAIGHT FUNCTION EVALUATION  ( MAIN+FEAS ONLY)
000216   200   CONTINUE
000216         IF (M.EQ.0) GO TO 220
000217         DO 210 I=1,M
000221         CALL RESTNX (I,RJ(I))
000223   210   CONTINUE
000226     220 CALL RESTNX (0,F)
         C EQUALITY CONSTRAINTS
000230         IF (MZ) 250,250,230
000232   230   DO 240 I=1,MZ
000234         KZ=M+I
000236     240 CALL RESTNX ( KZ,RJ(KZ))
000243   250   RETURN
         C CONSTRAINTS VIOLATED NOT SO BEFORE
000244   260   NSATIS=3
000245         P0=10.E35
000247         RETURN
000247         END
```

```
                SUBROUTINE FEASS
            C
            C              OCTOBER 1970
            C
            C FEAS DETERMINES WHETHER THE STARTING POINT IS FEASABLE. IF IT IS NOT,
            C FEAS LOOKS FOR A FEASABLE ONE. IF NONE EXISTS, A MESSAGE IS PRINTED
            C AND CONTROL RETURNS TO MAIN.
000002          COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000002          COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002          COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000002          COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,
               1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
               2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
               3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000002          NPHASE=1
000003          GO TO (10,50), NT2
000011      10  NFIX=1
000012          DO 30 I=1,N
000014          IF (X(I)) 20,20,30
000016      20  NFIX=2
000017          X(I)=1.E-05
000021      30  CONTINUE
000024          GO TO (50,40), NFIX
000032      40  NPHASE=4
000033          CALL EVALUS
            C JUST GET ALL CONSTRAINTS EVALUATED
000034          NPHASE=1
000035          WRITE (6,130)
000041          CALL OUTPUX( 2)
000043      50  IF (M) 90,90,60
000045      60  DO 70 I=1,M
000047          IF (RJ(I)) 100,100,70
000051      70  CONTINUE
000054      80  CALL TIMECS
000055          WRITE (6,140)
000061          G=0.0
000062          CALL RESTNX ( 0,F)
000064          CALL OUTPUX (2)
000066      90  RETURN
000067     100  CALL BODYS
000070          DO 110 I=1,M
000072          IF (RJ(I)) 120,120,110
000074     110  CONTINUE
000077          GO TO 80
000077     120  WRITE (6,150)                           120 CONTINUE
            C TO INDICATE TO MAIN TO START ON NEXT PROBLEM.
000103          NPHASE=5
000104          GO TO 90
            C
000105     130  FORMAT (1H0,2X,48HMADE VIOLATED NON-NEGATIVITIES SLIGHTLY POSITIVE
               1)
000105     140  FORMAT (51H0*****THE FEASIBLE STARTING POINT TO BE USED IS ...)
000105     150  FORMAT (3X,89HTHIS PROBLEM POSSESSES NO FEASIBLE STARTING POINT, W
               1ILL LOOK FOR DATA FOR NEXT PROBLEM.  )
000105          END
```

```
                SUBROUTINE FINALS(N2)
         C
         C               OCTOBER 1970
         C
         C FINAL CONTAINS THE TESTS USED TO DETERMINE WHETHER A POINT SATISFIES
         C THE FINAL CONVERGENCE CRITERION CHOOSEN TO DETERMINE IF THE NLP
         C PROBLEM HAS BEEN SOLVED.
         C N2 SET EQUAL TO 1 IF CONVERGENCE CRITERION IS SATISFIED.
         C N2 SET EQUAL TO 2 OTHERWISE.
000003          COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000003          COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000003          COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000003          COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,
               1 RSIG1, G1,     X1(100), X2(100), X3(100), XR2(100), XRI(100), PR1,
               2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
               3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000003          GO TO (10,20,30), NT5
000012   10     EPSIL=ABS(F/G-1.)
000016          IF (EPSIL-THETA0) 50,50,70
000021   20     IF (ABS(RSIGMA)-THETA0) 50,50,70
000025   30     IF (NUMINI-1) 50,40,40
000030   40     PEST=PR1-(PRI-P0)/(1.-1./SQRT(RATIO))
000040          EPSIL=ABS(PEST/G-1.)
000043          IF (EPSIL-THETA0) 50,70,70
000046   50     N2=1
000047          GO TO (80,60), NT6
000055   60     CALL PUNCHS
000056          GO TO 80
000060   70     N2=2
000061   80     RETURN
000062          END
```

```
              SUBROUTINE GRADS(IS)
        C
        C             OCTOBER 1970
        C
        C GRAD COMPUTES THE GRADIENT OF THE PENALTY FUNCTION AND THE OUTER
        C PRODUCT FACTORS OF THE MATRIX OF SECOND PARTIALS OF P.
        C  IF  (IS=1) ACCUM. MATRIX OF 2ND PARTIALS  IF(IS=2) OUNT
000003        COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NPI,NM1
000003        COMMON /EQALS/H, H1, MZ
000003        COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000003        COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000003        COMMON/CRSTS/OELX(100), OELX0(100), RHOIN,RATIO, EPSI, THETA0,
             1 RSIG1, G1,     X1(100), X2(100), X3(100), XR2(100), XRI(100), PR1,
             2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), OIAG(100),
             3 PREV3,AOELX, NTCTR, NUMINI, NPHASE, NSATIS
000003        GO TO (I0,30), IS
00001I  I0    DO 20 I=I,N
000013        DO 20 J=1,I
000014  20    A(I,J)=0.
000024  30    00 40 I=1,N
000026  40    OELX0(I)=0.
        C THIS SECTION WORKS CORRECTLY IN FEASIBILITY PHASE AS WELL AS NORMAL PH
00003I        GO TO (50,80), NT2
000037  50    00 70 I=1,N
000041        OELX0(I)=-RHO/X(I)
000044        GO TO (60,70), IS
000051  60    A(I,I)=(-OELX0(I)/X(I))
000056  70    CONTINUE
00006I  80    CONTINUE
00006I        IF (M.LE.0) GO TO I80
000063        00 I70 K=I,M
000064        CALL GRAOIS(K)
000065        IF (RJ(K).GI.0.0) GO TO 110
        C ALL VIOLATEO CONSTRAINT  GRAOS AOOEO TO OBJ. FUNCTION
00007I        DO 100 I=I,N
000072        IF (DEL(I)) 90,100,90
000073  90    OELX0(I)=OELX0(I)-OEL(I)
000076  100   CONTINUE
000101        GO TO 170
000101  I10   TT=RHO/RJ(K)
000104        00 160 I=1,N
000105        IF (DEL(I)) 120,160,120
        C IF OEL(I)=0 SKIP ALL THE FOLLOWING COMPUTATION INVOLVING * BY OEL(I)
000106  120   T=TT*OEL(I)
00011I        DELX0(I)=DELX0(I)-T
000113        GO TO (130,160), IS
000120  130   T=T/RJ(K)
000122        DO 150 JJ=I,I
000124        IF (OEL(JJ)) 140,150,140
000125  140   A(I,JJ)=A(I,JJ)+T*OEL(JJ)
000133  150   CONTINUE
000136  160   CONTINUE
00014I  170   CONTINUE
        C  EQUALITY CHANGES FOR GRAD
000144  180   IF (MZ.LE.0) GO TO 250
000146        GO TO (250,190,250), NPHASE
000155  I90   RQ=2./RHO
```

```
000157          DO 240 J=1,MZ
000161          K=M*J
000163          CALL GRADIS(K)
000164          TT=RQ*RJ(K)
000167          DO 230 I=1,N
000171          IF (DEL(I).EQ.0.0) GO TO 230
000172          DELX0(I)=DELX0(I)+DEL(I)*TT
000175          GO TO (200,230), IS
000203    200   T=RQ*DEL(I)
000206          DO 220 JJ=I,I
000207          IF (DEL(JJ)) 210,220,210
000210    210   A(I,JJ)=A(I,JJ)+T*DEL(JJ)
000216    220   CONTINUE
000221    230   CONTINUE
000224    240   CONTINUE
000226    250   GO TO (260,280), IS
000234    260   DO 270 I=I,N
000236    270   DIAG(I)=A(I,I)
000246    280   GO TO (290,330,290), NPHASE
          C LEAVES NEGATIVE GRADIENT IN DELP
000255    290   DO 300 I=I,N
000257    300   DELX0(I)=-DELX0(I)
000263    310   ADELX=0.
000264          DO 320 I=1,N
000266    320   ADELX=ADELX+DELX0(I)**2
000273          ADELX=SQRT(ADELX)
000275          RETURN
000275    330   CALL GRADIS(0)
000277          DO 340 I=1,N
000302    340   DELX0(I)=-DELX0(I)-DEL(I)
          C LEAVES THE NEG. GRAD OF P IN DELX0
000307          GO TO 310
000307          END
```

```
             SUBROUTINE INVERX(NSME)
       C
       C              OCTOBER 1970
       C
       C INVERS SOLVES THE SET OF EQUATIONS FOR THE MOVE-VECTOR USING THE
       C CROUT PROCEDURE. IF THE MATRIX IS NOT POSITIVE DEFINITE, A DIFFERENT
       C METHOD IS USED.
       C
       C****PERFORMING A L-U DECOMPOSITION OF THE MATRIX A, TAKING ADVANAGE OF
       C****THE SYMMETRY OF THE A MATRIX.
       C****IF A NON-POSITIVE PIVOT CANDIDATE IS GENERATED, THEN MCCORMICK,S
       C****PROCEDURE IS USED(SEE PP. 167-168 IN FIACCO AND MCCORMICK).
       C
       C****IF NSME =1 WORKING WITH A NEW A MATRIX,  IF NSME= 2 USING PREVIOUS
       C****A MATRIX, BUT HAVE A NEW RIGHT-HAND-SIDE.
       C
       C****NINV IS THE NUMBER OF NON_POSITIVE PIVOT CANDIDATES GENERATED.
000003       COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000003       COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000003       COMMON/CRSTS/DELX(100), OELXO(100), RHOIN,RATIO, EPSI, THETAO,
            1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
            2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
            3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000003       COMMON/EXPOKX / NEXOP1, NEXOP2, XEP1, XEP2
000003       DIMENSION B(100)
000003       EQUIVALENCE (B,DELX)
000003       GO TO(20, 170), NSME
000011 20    NINV=0
000012       IF (A(1,1)) 40,30,50
000014 30    NINV=1
000015       GO TO 70
000016 40    NINV=1
000017 50    A(1,1)=1./A(1,1)
000021       DO 60 I=2,N
000022 60    A(1,I)=A(1,I)*A(1,1)
000030 70    DO 160 J=2,N
000032       JM1=J-1
000034       T=0.
000035       DO 90 I=1,JM1
000036       IF (A(I,J)) 80,90,80
000041 80    T=T+A(J,I)*A(I,J)
000051 90    CONTINUE
000054       A(J,J)=A(J,J)-T
000060       IF (A(J,J)) 110,100,120
000064 100   NINV=NINV+1
000066       GO TO 170
000066 110   NINV=NINV+1
000070 120   A(J,J)=1./A(J,J)
000074       IF (J.EQ.N) GO TO 170
000076       JP1=J+1
000100       DO 150 L=JP1,N
000101       T=0.
000102       DO 140 I=1,JM1
000104       IF (A(I,J)) 130,140,130
000107 130   T=T+A(L,I)*A(I,J)
000117 140   CONTINUE
000122       A(L,J)=A(L,J)-T
```

```
000127            A(J,L)=A(L,J)*A(J,J)
000136      150   CONTINUE
000140      160   CONTINUE
000142      170   CONTINUE
000142            IF (NINV) 180,180,290
000144      180   B(1)=B(1)*A(1,1)
000146            DO 210 J=2,N
000147            T=0.
000150            JM1=J-1
000152            DO 200 I=1,JM1
000153            IF (A(J,I)) 190,200,190
000156      190   T=T+A(J,I)*B(I)
000164      200   CONTINUE
000167            B(J)=(B(J)-T)*A(J,J)
000174      210   CONTINUE
000177            DO 240 I=1,NM1
000200            NMK=N-I
000202            DO 230 J=1,I
000203            L=NP1-J
000205            IF (A(NMK,L)) 220,230,220
000210      220   B(NMK)=B(NMK)-A(NMK,L)*B(L)
000216      230   CONTINUE
000221      240   CONTINUE
000223      250   GO TO (280,260), NT3
000231      260   WRITE (6,430)                                   260 CONTINUE
000235            WRITE (6,420) (DELX0(I),I=1,N)
000250      270   WRITE (6,440)                                   270 CONTINUE
000254            WRITE (6,420) (DELX(I),I=1,N)
000267      280   RETURN
            C---  COMPUTE ORTHOGONAL MOVE
000270      290   CONTINUE
000270            DO 350 II=1,N
000273            I=N-II+1
000276            IF (A(I,I)) 310,300,320
000302      300   B(I)=0.0
000304            GO TO 350
000304      310   B(I)=1.0
000306            GO TO 330
000307      320   B(I)=0.0
000311      330   IP1=I+1
000313            IF (IP1.GT.N) GO TO 350
000316            DO 340 J=IP1,N
000317      340   B(I)=B(I)-A(I,J)*B(J)
000331      350   CONTINUE
000334            GO TO 360
            C-- CHECK MAYBE DO DIFF FOR P.S.D.
000334      360   ZC2=0.0
000335            DO 370 I=1,N
000337      370   ZC2=ZC2+DELX0(I)*B(I)
000344            IF (ZC2) 380,400,400
000345      380   DO 390 I=1,N
000347      390   B(I)=-B(I)
000353      400   WRITE (6,450)
            C MCC ZANGWILL  ONE MOD
000357            IF (NEXOP2.NE.2) GO TO 250
000362            DO 410 K=1,N
000364      410   B(K)=B(K)+DELX0(K)
000371            GO TO 250
```

```
       C
000371 420    FORMAT (7E17.8)
000371 430    FORMAT (1H0,6X,12HDEL P VECTOR)
000371 440     FORMAT(1H0,6X,24HSECOND ORDER MOVE VECTOR)
000371 450    FORMAT (1H0,6X,15HORTHOGONAL MOVE)
000371        END
```

```
                    SUBROUTINE OPTS
             C
             C              MARCH 1971
             C
             C OPT LOOKS FOR A MINIMUM ALONG THE SEARCH VECTOR USING THE GOLOEN
             C SECTION SEARCH METHOD.
000002              COMMON/SHARES/X(100), OEL(100), A(100,100),N,M, MN,NP1,NM1
000002              COMMON /VALDES/F,G,P0,RSIGMA, RJ(200), RHO
000002              COMMON/CRSTS/DELX(100), OELX0(100), RHO1N,RAT10, EPSI, THETA0,
                  1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
                  2 PR2,P1, F1, RJ1(200), DOTT, PGRA0(100), DIAG(100),
                  3 PREV3,AOELX, NTCTR, NUMIN1, NPHASE, NSATIS
000002              KSW=1
000003              N405=1
000004              P31=P0
000006              ISW=1
000007              DOTT=0.
000010              DO 10 J=1,N
000011       10     DOTT=DOTT+OELX(J)*DELX0(J)
000016              GO TO 40
000017       20     DO 30 I=1,N
000021       30     OELX(I)=-DELX(I)
000025       40     CONTINUE
000025              N404=0
000026              MN=MN+1
             C MN IS NOW NUMB. OF POINTS AFTER MIN ACHIEVEO
000030              NTCTR=NTCTR+1
000031              DO 50 I=1,N
000032       50     X2(I)=X(I)
000036              PX1=P0
000037              N401=0
000040       60     N401=N401+1
000042              DO 70 I=1,N
000043       70     X(I)=X2(I)+OELX(I)
000050              CALL EVALUS
             C 1 MEANS SATIS.OF CONSTRAINT NT.PREV. 2MEANS NOCHANGE 3MEANS VIOLATION
             C IF POINT IS NOT FEASIBLE GIVE IT AN ARBRITRARILY HIGH VALUE
000051              GO TO (540,90,80), NSATIS
000060       80     PX2=10.E35
000062              P0=10.E35
000063              GO TO 100
000064       90     CONTINUE
000064              PX2=P0
000066              IF (PX1-PX2) 100,100,150
000070       100    IF (N401-2) 130,110,110
000073       110    DO 120 I=1,N
000075       120    X1(I)=X(I)
000101              P1=PX2
000102              GO TO 430
             C ONLY ONE  POINT  SO FAR COMPUTED
000103       130    DO 140 I=1,N
000105       140    X3(I)=X2(I)
000111              PREV3=PX1
000112              GO TO 180
000113       150    DO 160 I=1,N
000115              X3(I)=X2(I)
000117              X2(I)=X(I)
```

```
000120   160   DELX(I)=1.61803399*DELX(I)
000124         PREV3=PX1
000126         PX1=PX2
000127         GO TO 60
               C GOLDEN SECTION SEARCH METHOD.
               C B VECTOR  GOES TO X1(I)
000127   170   P0=1.E36
000131         N404=N404+1
000132   180   DO 190 I=1,N
000134   190   X1(I)=X(I)
000140         P1=P0
000141         DO 200 I=1,N
000143         X(I)=.38196601*(X1(I)-X3(I))+X3(I)
000150   200   X2(I)=X(I)
000153         CALL EVALUS
000154         GO TO (540,270,210), NSATIS
000163   210   IF (N404.LT.30) GO TO 170
000166   211   CONTINUE
               C THERE IS NO REFERENCE TO 211. THE ABOVE STATEMENT IS A DUMMY STATEMENT
               C--IT IS POSSIBLE NO FEASIBLE POINT EXIST. IF NOT TRY MOVING ON DELXO.
               C--  IF IT IS NOT POSSIBLE TO MOVE ON DELXO THEN WE MUST BE AT A
               C--  SOLUTION OF NLP PROBLEM.
000166         IF (N404.GT.100) GO TO 240
000172   220   DO 230 I=1,N
000174         IF (ABS(ABS(X3(I)/X1(I))-1.).GT.1.E-7) GO TO 170
000203   230   CONTINUE
000205   240   GO TO (250,260), N405
000213   250   N405=2
               C--TRY TO MOVE ON GRADIENT.
000214         NTCTR=NTCTR-1
000216         MN=MN-1
000217         GO TO 20
000217   260   WRITE (6,580)                    260 CONTINUE
000223         CALL TIMECS
000224         CALL OUTPUX(1)
000226         CALL  REJECA
000227         STOP 22042
               C
000231   270   CONTINUE
000231         N404=0
000232         PX1=P0
000234         DO 280 I=1,N
000235   280   X(I)=0.381966O1*(X1(I)-X2(I))+X2(I)
000244         CALL EVALUS
000245         GO TO (540,290,220), NSATIS
000254   290   PX2=P0
000256         N401=1
000257   300   N401=N401+1
000261         IF (N401-25) 340,310,310
000263   310   KSW=2
000264         IF (N401-40) 320,460,460
000267   320   DO 330 I=1,N
000271         IF (ABS(X2(I)/X(I)-1.0).GE.1.E-7) GO TO 340
000276   330   CONTINUE
000301         GO TO 460
000301   340   IF (ABS(PX1/PX2-1.).LE.1.E-7) GO TO 460
000306         IF (PX1-PX2) 350,460,400
               C FROM LEFTORIGHT  X3(I)(PREV3)X2(I)(PX1)X(I)PX2 X1(I)P1
```

```
000310   350   DO 360 I=I,N
000312   360   X1(I)=X(I)
         C THROW AWAY RIGHT PART
000316         P1=PX2
000317         DO 370 I=1,N
         C POINTXP1 BECOMES XP2
00032I   370   X(I)=.38196601*(X1(I)-X3(I))+X3(I)
         C TEMPORARILY IN  X STORAGE
000330         CALL EVALUS
00033I         GO TO (540,380,170), NSATIS
000340   380   CONTINUE
000340         PX2=PXI
         C SWITCH VECTORS TO PROPER POSITION
000342         PXI=P0
000343         DO 390 I=I,N
000344         XX=X2(I)
000346         X2(I)=X(I)
000347   390   X(I)=XX
000353         GO TO 300
         C LEFT SIDE  TOSSED  AWAY
         C-- CHANGES FOR NONUNIMODAL FN
         C-- GO TO THROW AWAY RIGHT IN THIS CASEINIT VAL LT FIB PT
000353   400   IF (PREV3-PX2) 350,350,4I0
000356   410   DO 420 I=1,N
000360         X3(I)=X2(I)
000362   420   X2(I)=X(I)
000365         PREV3=PX1
000367         PXI=PX2
000370   430   DO 440 I=I,N
000372   440   X(I)=0.38I96601*(XI(I)-X2(I))+X2(I)
00040I         CALL EVALUS
000402         GO TO (540,450,170), NSATIS
0004II   450   CONTINUE
0004II         PX2=P0
000413         GO TO 300
         C THE INTERIOR POINTS NOW GIVE EQUAL VALUE FOR P, COMPUTE MIDPOINT.
000413   460   DO 470 I=I,N
000415         DELX0(I)=X(I)
000417         X(I)=(DELX0(I)+X2(I))*0.5
000422   470   CONTINUE
000424         CALL EVALUS
000425         GO TO (480,490), KSW
000433   480   IF (ABS(P0/PX1-1.).GT.1.E-7) GO TO 520
00044I   490   GO TO (500,5I0), ISW
000447   500   IF (P0.LT.P3I) GO TO 510
000452         ISW=2
         C IF P-FUNCTION DION,T  GO DOWN TRY NEG VECT.
000453         GO TO 20
000453   510   RETURN
000454   520   DO 530 I=I,N
000456   530   X(I)=DELX0(I)
000462         GO TO 350
         C ARE  WE NOW IN FEASIBILITY PHASE
000462   540   DO 550 I=I,M
000464         IF (RJ(I)) 560,560,550
000466   550   CONTINUE
00047I         NSATIS=4
000472         RETURN
```

```
       C---   PROBLEM HAS BECOME FEASIBLE
       C ---   P - FUNCTION CHANGES IF A CONSTRAINT BECOMES FEASIBLE
000472 560    MN=0
000473        DO 570 I=1,M
000475 570    RJ1(I)=RJ(I)
000501        RETURN
       C
000501 580    FORMAT ( 80H  OPT CAN*T FIND A FEASIBLE POINT,THAT GIVES A LOWER V
             1ALUE OF THE P-FUNCTION.         )
000501        END
```

```
                  SUBROUTINE OUTPUX (K)
         C
         C                OCTOBER 1970
         C
         C OUTPUT PRINTS OUT INFORMATION ON THE RESULTS OF EACH ITERATION AND THE
         C SOLUTION ESTIMATES AND THE ESTIMATES OF THE LAGRANGE MULTIPLIERS
000003            COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000003            COMMON /EQALS/H, H1, MZ
000003            COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000003            COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000003            COMMON/CRSTS/DELX(100), OELX0(100), RHOIN,RATIO, EPS1, THETA0,
                 1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
                 2 PR2,P1, F1, RJ1(200), DOTT, PGRA0(100), DIAG(100),
                 3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000003            NZ=M+MZ
000005            GO TO (10,20), K
000013   10       WRITE (6,60)
000017            WRITE (6,70) NTCTR,DOTT,RHO,ADELX,NPHASE
000035   20       WRITE (6,80) F,P0,G,RSIGMA,H
000053            WRITE (6,90) (X(I),I=1,N)
000066            WRITE (6,110)
000072            GO TO (30,40), NT2
000101   30       WRITE (6,120)
000105            WRITE (6,100) (RJ(1),I=1,NZ)
000120            GO TO 50
000122   40       WRITE (6,100) (RJ(I),I=1,NZ)
000135   50       RETURN
         C
000136   60       FORMAT (50H0*********************************************        )
000136   70       FORMAT (10X,6HPOINT=I4,6X,6H DOTT=E15.7,6X,4HRHO=E15.7,6X,10HMAGNI
                 1TUDE=E15.7,9X,6HPHASE=12)
000136   80       FORMAT (8X,2HF=E15.7,5X,2HP=E15.7,5X,2HG=E15.7,5X,7HRSIGMA=E15.7,5
                 1X,2HH=E15.7)
000136   90       FORMAT (6X,25HTHE CURRENT VALUE OF X IS/(6E20.7))
000136   100      FORMAT (6E20.7)
000136   110      FORMAT (6X,21HTHE CONSTRAINT VALUES)
000136   120      FORMAT (28X,34HNOT INCLUDING THE NON-NEGATIVITIES)
000136            END
```

```
          SUBROUTINE OUTPUX (K)
9999      CONTINUE
          RETURN
          END
```

```
                 SUBROUTINE PEVALX
              C
              C            OCTOBER 1970
              C
              C PEVALU COMPUTES THE VALUE OF THE PENALTY FUNCTION ANO THE VALUE OF THE
              C DUAL USING PREVIOUSLY COMPUTEO VALUES FOR F, AND RJ.
000002           COMMON/SHARES/X(100), OEL(100), A(100,100),N,M, MN,NP1,NM1
000002           COMMON /EQALS/H, H1, MZ
000002           COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002           COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000002           COMMON/CRSTS/OELX(100), OELX0(100), RHOIN,RAT10, EPS1, THETA0,
                1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
                2 PR2,P1, F1, RJ1(200), OOTT, PGRAO(100), OIAG(100),
                3 PREV3,AOELX, NTCTR, NUMIN1, NPHASE, NSATIS
000002           H=0.0
000003           RSIGMA=0.0
              C NONNEGS IF INCLUDED ARE AODEO TO  P--ARE POS, IN ALL PHASES
000004           GO TO (10,30), NT2
000012        10 00 20 I=1,N
000014        20 RSIGMA=RSIGMA-RHO*ALOG(X(I))
000024        30 GO TO (40,50,150), NPHASE
              C OBJECTIVE FUNCTION   - SIGMA VIOL. CONSTS.
000033        40 F=0.0
000034        50 IF (M) 100,100,60
000036        60 00 90 J=1,M
000040           IF (RJ(J)) 80,80,70
000042        70 RSIGMA=RSIGMA-RHO*ALOG(RJ(J))
000047           GO TO 90
000050        80 F=F-RJ(J)
000053        90 CONTINUE
              C EQUALITIES NOT AOOED IN FEAS. PHASE
000056       100 CONTINUE
000056           IF (MZ) 140,140,110
000060       110 GO TO (140,120,150), NPHASE
000067       120 00 130 I=1,MZ
000071           K=M+I
000073       130 H=H+RJ(K)**2
000100           H=H/RHO
000101       140 HS=H+RSIGMA
000103           P0=F+HS
000105           HMS=2.*H-RHO*FLOAT(M)
000111           G=F+HMS
000113           IF (NT2.EQ.1) G=G-RHO*FLOAT(N)
000120       150 RETURN
000121           END
```

```
              SUBROUTINE PUNCHS
        C
        C               OCTOBER 1970
        C
        C THIS SUBROUTINE PUNCHES THE STOPPING POINTS AND ASSOCIATED PARAMETERS
        C SO THAT ANOTHER RUN MAY BE MADE STARTING WHERE THE CURRENT ONE
        C STOPPED
        C  THIS ROUTINE IS CALLED IF NT6=2.
000002        COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000002        COMMON /EQALS/H, H1, MZ
000002        COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002        COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000002        COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,
             1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
             2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
             3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000002        COMMON/EXPOPX / NEXOP1, NEXOP2, XEP1, XEP2
000002        T=60.0
000004        WRITE (7,10) EPSI,RHO,THETA0,RATIO,T,M,N,MZ
        C  TMMAX IS SET TO 60. SECONDS
000027        NT1=3
000030        WRITE (7,20) (X(I),I=1,N)
        C SET  RHO OPTION SO THIS VALUE OF RHO WILL BE USE FOR THE RESTART.
000043        WRITE (7,30) NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000073        WRITE (7,20) XEP1, XEP2
000103        WRITE (7,30) NEXOP1,NEXOP2
000113        RETURN
        C
000114  10    FORMAT (5E12.5,3I4)
000114  20    FORMAT (6E12.5)
000114  30    FORMAT (10I7)
000114        END
```

```
                   SUBROUTINE REJECX
             C
             C           OCTOBER 1970
             C
             C REJECT RETURNS THE STORED VALUES OF THE OBJECTIVE FUNCTION, THE
             C CONSTRAINT FUNCTIONS AND THE PENALTY FUNCTION TO THEIR NORMAL LOCATION
000002             COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000002             COMMON /EQALS/H, H1, MZ
000002             COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000002             COMMON/CRSTS/DELX(100), DELX0(100), RHO1N,RATIO, EPS1, THETA0,
                 1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
                 2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
                 3 PREV3,ADELX, NTCTR, NUMIN1, NPHASE, NSATIS
000002             DO 10 I=1,N
000004       10    X(I)=X1(I)
000010             MMZ=M+MZ
000012             DO 20 J=1,MMZ
000013       20    RJ(J)=RJ1(J)
000017             P0=P1
000020             RSIGMA=RSIG1
000022             G=G1
000023             F=F1
000025             H=H1
000026             RETURN
000027             END
```

```
              SUBROUTINE RHOCOX
          C
          C             OCTOBER 1970
          C
          C   SUBROUTINE  TO COMPUTE INITIAL RHO VALUE
          C   CONTROLLED  BY  COL. 7  ON OPTION CARD
000002        COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000002        COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002        COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000002        COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPS1, THETA0,
             1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
             2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
             3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000002        GO TO (110,50,10,190), NT1
000012   10   RHO=RHOIN
000014   20   IF (RHO) 30,30,40
000016   30   RHO=1.
000020   40   RETURN
000021   50   NPAR1=1
000022   60   RHO=1.
          C   2 MEANS RHO WHICH  MINIMIZES GRADIENT  MAG.
000024        CALL GRADS(2)
000025        DO 70 I=1,N
000027   70   PGRAD(I)=DELX0(I)
000033        RHO=2.
000034        CALL GRADS(2)
000036        DO 80 I=1,N
000040        DELX0(I)=DELX0(I)-PGRAD(I)
000042   80   PGRAD(I)=PGRAD(I)-DELX0(I)
000046        GO TO (90,130), NPAR1
000054   90   DOT1=0.
000055        DOT2=0.
000056        DO 100 I=1,N
000057        DOT1=DOT1+DELX0(I)*PGRAD(I)
000062   100  DOT2=DOT2+DELX0(I)**2
000067        RHO=ABS(DOT1/DOT2)
000071        GO TO 20
          C  3 MEANS COMPUTE RHO SO AS TO MINIMIZE DEL P(/DDP/1.)DEL P
000072   110  NPAR2=1
000073   120  NPAR1=2
          C   USE  OF  AND OR  SUBROUTINE
000074        GO TO 60
000075   130  RHO=1.
          C   ASSUME   SIGMA TERM IS CONSID. GRTER THAN  F TERM
000077        CALL SECORX (2)
000100        DO 140 I=1,N
000102   140  DELX(I)=PGRAD(I)
000106        CALL INVERX (1)
000107        DO 150  I=1,N
000111        X1(I)=DELX(I)
000113   150  DELX(I)=DELX0(I)
000116        CALL SECORX (2)
000120        CALL INVERX (1)
000122        DO 160  I=1,N
000124   160  XR2(I)=DELX(I)
000130        GO TO (170,200), NPAR2
000136   170  DOT1=0.
```

```
000137             DOT2=0.
000140             DO 180 I=1,N
000141             DOT1=DOT1+PGRAD(I)*X1(I)
000144      180    DOT2=DOT2+DELX0(I)*XR2(I)
000151             RHO=SQRT(ABS(DOT1/DOT2))
000156             GO TO 20
000156      190    NPAR2=2
            C      RHO MINIMIZES  2ND ORDER MOVE
000157             GO TO 120
            C      USES  INTERNAL  SUB. TO COM  /DDP/=1 OF AND  /DDP/= OR
000160      200    DOT1=0.0
000161             DOT2=0.0
000162             DO 210 I=1,N
000163             DOT1=X1(I)**2+DOT1
000166      210    DOT2=X1(I)*XR2(I)+DOT2
000172             RHO=ABS(DOT1/DOT2)
000175             GO TO 20
000175             END
```

```
              SUBROUTINE SECORX(IS)
        C
        C              OCTOBER 1970
        C
        C SECORD EVALUATES THE MATRIX OF SECOND PARTIALS OF THE PENALTY
        C FUNCTION.
        C
        C- (1) MEANS DONT   COMPUTE GRAD, OUTER PRODUCT(IN SECOND)
000003        COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000003        COMMON /EQALS/H, H1, MZ
000003        COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000003        COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000003        COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPS1, THETA0,
             1 RSIG1, G1,     X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
             2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
             3 PREV3,ADELX, NTCTR, NUMIN1, NPHASE, NSATIS
000003        DO 10 I=1,N
000005        DO 10 J=I,N
000006  10    A(I,J)=0.
000017        GO TO (230,20), IS
        CGRAD. TERM NOT PREV. COMPUTED
000024  20    DO 30 I=1,N
000026        DO 30 J=1,I
000027        A(I,J)=0.0
000033  30    CONTINUE
000037        GO TO (40,60), NT2
000045  40    DO 50 I=1,N
000047  50    A(I,I)=RHO/X(I)**2
000060  60    CONTINUE
000060        IF (M.LE.0) GO TO 130
000062        DO 120  IN=1,M
000063        IF (RJ(IN)) 120,120,70
000065  70    CALL GRADIS(IN)
000067        TT = RHO/RJ(IN)**2
000072        DO 110  I=1,N
000074        IF (DEL(I)) 80,110,80
000075  80    T=TT*DEL(I)
000100        DO 100 J=1,I
000101        IF (DEL(J)) 90,100,90
000102  90    A(I,J)=A(I,J)+T*DEL(J)
000110  100   CONTINUE
000113  110   CONTINUE
000116  120   CONTINUE
        C EQUALITY  CONSTRAINTS
000121  130   IF (MZ) 210,210,140
000123  140   GO TO (210,150,230), NPHASE
000132  150   RQ=2./RHO
000134        DO 200 JJ=1,MZ
000136        IN=M+JJ
000140        CALL GRADIS(IN)
000141        DO 190  I=1,N
000144        IF (DEL(I)) 160,190,160
000145  160   T=RQ*DEL(I)
000150        DO 180 J=1,I
000151        IF (DEL(J)) 170,180,170
000152  170   A(I,J)=A(I,J)+T*DEL(J)
000160  180   CONTINUE
```

```
000163   190  CONTINUE
000166   200  CONTINUE
000170   210  DO 220 I=1,N
000172        DIAG(I)=A(I,I)
000176   220  A(I,I)=0.
         C  READY NOW FOR MATRIX OF 2ND PARTIALS OF RESTRAINTS
000203   230  GO TO (240,510,520), NT10
000212   240  IF (M.LE.0) GO TO 340
000214        DO 330 IN=1,M
000215        LORN=2
         C CONSTRAINT ASSUMED NONLINEAR
000216        CALL MATRXX (IN,LORN)
000220        IF (LORN.LT.2) GO TO 330
000224        IF (RJ(IN) .GT. 0.0) GO TO 280
000227        DO 260 I=2,N
000230        IM1=I-1
000232        DO 260 J=1,IM1
000233        IF (A(J,I)) 250,260,250
000236   250  A(I,J)=A(I,J)-A(J,I)
000245        A(J,I)=0.
000247   260  CONTINUE
000254        DO 270 I=1,N
000256        DIAG(I)=DIAG(I)-A(I,I)
000262   270  A(I,I)=0.0
000270        GO TO 330
000270   280  T=-RHO/RJ(IN)
000273        DO 300 I=2,N
000274        IM1=I-1
000276        DO 300 J=1,IM1
000277        IF (A(J,I)) 290,300,290
000302   290  A(I,J)=A(I,J)+T*A(J,I)
000312        A(J,I)=0.
000315   300  CONTINUE
000322        DO 320 I=1,N
000324        IF (A(I,I)) 310,320,310
000327   310  DIAG(I)=DIAG(I)+T*A(I,I)
000335        A(I,I)=0.
000340   320  CONTINUE
000343   330  CONTINUE
000346   340  CONTINUE
000346        GO TO (520,350,520), NPHASE
000355   350  IF (MZ.EQ.0) GO TO 420
         C--   EQUALITY  SECOND PARTIALS HERE
000356        IF (NT10.GE.2) GO TO 420
000361        DO 410 II=1,MZ
000362        IN=M+II
000364        LORN=2
000365        CALL MATRXX (IN,LORN)
000367        IF (LORN.LT.2) GO TO 410
000373        T=2.*RJ(IN)/RHO
000376        DO 380 I=2,N
000377        IM1=I-1
000401        DO 370 J=1,IM1
000402        IF (A(J,I)) 360,370,360
000405   360  A(I,J)=A(I,J)+T*A(J,I)
000415        A(J,I)=0.0
000420   370  CONTINUE
000423   380  CONTINUE
```

```
000425          DO 400 I=1,N
000427          IF (A(I,I)) 390,400,390
000432   390    DIAG(I)=DIAG(I)+T*A(I,I)
000440          A(I,I)=0.0
000443   400    CONTINUE
000446   410    CONTINUE
         C  GET MATRIX OF 2ND PARTIALS OF OBJECTIVE FUNCTION
000451   420    LLL=2
000452          CALL MATRXX( 0,LLL)
000454          IF (LLL.LT.5) GO TO 490
000460          DO 440 I=2,N
000461          IM1=I-1
000463          DO 440 J=1,IM1
000464          IF (A(J,I)) 430,440,430
000467   430    A(I,J)=A(I,J)+A(J,I)
000476   440    A(J,I)=A(I,J)
000511          DO 470 I=1,N
000513          IF (A(I,I)) 450,460,450
000516   450    A(I,I)=DIAG(I)+A(I,I)
000522          GO TO 470
000523   460    A(I,I)=DIAG(I)
000527   470    CONTINUE
000532   480    RETURN
000533   490    DO 500 I=1,N
000535          A(I,I)=DIAG(I)
000541          DO 500 J=1,N
000542   500    A(I,J)=A(J,I)
000556          GO TO 480
000556   510    GO TO (520,350,350), NPHASE
000565   520    DO 530 I=2,N
000567          IM1=I-1
000571          DO 530 J=1,IM1
000572   530    A(J,I)=A(I,J)
000606          DO 540 I=1,N
000607   540    A(I,I)=DIAG(I)
000616          GO TO 480
000617          END
```

```
              SUBROUTINE SETS(TMMAX)
        C
        C     FEBUARY 1971
        C
        C SET STORES THE TIME AT WHICH THE PROBLEM IS BEGUN
000003        COMMON /TSWS/NSWW
000003        COMMON /TMXS/TMO,EXT,EXT90
        C
        C  SECOND GIVES JOB CPU EXECUTION TIME IN 1/1000 OF A SECOND
        C
000003        CALL SECOND (TMO)
000005        EXT=TMMAX+TMO
000007        EXT90= TMO + 0.90*TMMAX
000012        NSWW=1
000013        RETURN
000014        END
```

```
      SUBROUTINE STORES
C
C              OCTOBER 1970
C
C STORE STORES THE VALUES OF THE CURRENT POINT AND THE ASSOCIATED
C VALUES OF THE FUNCTIONS IN A TEMPORARY AREA.
000002      COMMON/SHARES/X(100), DEL(100), A(100,100),N,M, MN,NP1,NM1
000002      COMMON /EQALS/H, H1, MZ
000002      COMMON /VALUES/F,G,P0,RSIGMA, RJ(200), RHO
000002      COMMON/CRSTS/DELX(100), DELX0(100), RHOIN,RATIO, EPSI, THETA0,
           1 RSIG1, G1,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
           2 PR2,P1, F1, RJ1(200), DOTT, PGRAD(100), DIAG(100),
           3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000002      DO 10 I=1,N
000004   10 X1(I)=X(I)
000010      MMZ=M+MZ
000012      DO 20 J=1,MMZ
000013   20 RJ1(J)=RJ(J)
000017      P1=P0
000020      F1=F
000022      G1=G
000023      RSIG1=RSIGMA
000025      H1=H
000026      RETURN
000027      END
```

```
                 SUBROUTINE TCHECK
            C
            C              FEBUARY 1971
            C
            C TCHECK CHECKS THE NUMBER OF SECONDS THAT HAVE ELAPSED SINCE THE START
            C OF THE PROBLEM. IF THE SOLUTION IS TAKING LONGER THAN 90 PER-CENT
            C OF THE ESTIMATED MAXIMUM TIME, A SWITCH IS SET TO GIVE MORE OUTPUT.
000002           COMMON /OPTNSS/NT1,NT2,NT3,NT4,NT5,NT6,NT7,NT8,NT9,NT10
000002           COMMON /TSWS/NSWW
000002           COMMON /TMXS/TMO,EXT,EXT90
000002           CALL SECOND (SECS)
000004           IF (SECS.LT.EXT90) RETURN
            C GETTING CLOSE TO EXCEEDING THE TIME LIMIT SET OUTPUT OPTION TO GIVE
            C MORE OUTPUT.
000007           NT3=2
000010           X=SECS - TMO
000012           WRITE(6,10) X
000020      10   FORMAT (6X,5HTIME=,F9.3,8H SECONDS  )
000020           IF(SECS .GT. EXT) NSWW=2
000024           CALL OUTPUX (1)
000026           RETURN
000027           END
```

```
              SUBROUTINE TIMECS
        C
        C     FEBUARY 1971
        C
        C TIMEC CHECKS THE NUMBER OF SECONDS THAT HAVE ELAPSED SINCE THE START
        C OF THE PROBLEM. IT PRINTS THIS NUMBER. IF THE SOLUTION IS TAKING
        C TIMEC CHECKS THE NUMBER OF SECONDS THAT HAVE ELAPSED SINCE THE START
        C OF THE PROBLEM. IT PRINTS THIS NUMBER. IF THE SOLUTION IS TAKING
        C LONGER THAN THE ESTIMATED MAXIMUM TIME. A SWITCH IS SET TO TERMINATE
        C THE RUN.
000002        COMMON /TSWS/NSWW
000002        COMMON /TMXS/TMO,EXT,EXT90
        C
        C  SECOND GIVES JOB CPU EXECUTION TIME IN 1/1000 OF A SECOND
        C
000002        CALL SECOND (SECS)
000004        X=SECS-TMO
000006        WRITE (6,20) X
000014        IF (SECS.LT.EXT) GO TO 10
000017        NSWW=2
000020  10    RETURN
        C
000021  20    FORMAT (6X,5HTIME=F9.3,8H SECONDS)
000021        END
```

```
              SUBROUTINE XMOVES
         C
         C              MARCH 1971
         C
         C XMOVE DETERMINES THE VECTOR ALONG WHICH THE SEARCH FOR A MINIMUM IS
         C USING OPT.
000002         COMMON/SHARES/X(100), DFL(100), A(100,100),N,M, MN,NP1,NM1
000002         COMMON/CRSTS/DFLX(100), DFLX0(100), RHOIN,RATIO, EPS1, THETA0,
             1 RSIGI, GI,    X1(100), X2(100), X3(100), XR2(100), XR1(100), PR1,
             2 PR2,PI, FI, RJ1(200), DOTT, PGRAD(100), DIAG(100),
             3 PREV3,ADELX, NTCTR, NUMINI, NPHASE, NSATIS
000002         COMMON/EXPDMX / NEXOP1, NEXOP2, XEP1, XEP2
000002         COMMON /XVES/SIG(100),YY(100),XXX(100),DELL(100)
         C--NEXOP2 DETERMINES HOW MOVE IS TO BE MADE
         C NEXOP2 = 1   USE MODIFIED  NEWTON RAPHSON METHOD.
         C        = 2   USE MODIFIED  NEWTON RAPHSON METHOD, BUT ADD DELX0 TO
         C              ORTHOGONAL MOVE VECTOR IF HESSIAN IS INDEFFINITE.
         C        = 3   USE STEEPEST DESCENT METHOD.
         C        = 4   USE MCCORMICK'S MODIFICATION OF THE FLETCHER-POWELL
         C              METHOD.
000002         GO TO (10,10,180,30), NEXOP2
         C--NEWTON -RAPH WITH WHATEVER METHOD IS IN INVERSE
000012   10    CALL GRADS(1)
         C--ONE (1) MEANS ACCUMULATE MATRIX OF SECOND PARTIAL DERIVATIVES
000014         CALL SECDRX (1)
000016         DO 20 I=1,N
000020   20    DELX(I)=DELX0(I)
000024         CALL INVERX(1)
         C IF A NONPOSITVE PIVOT IS ENCOUNTERED IN INVERSE AN ATTEMPT IS MADE TO
         C COMPUTE A VECTOR HAVING A POSITIVE DOT PRODUCT WITH A NEGATIVE
         C EIGENVECTOR AND THE NEGATIVE OF DEL P.
000025         CALL STORES
000026         CALL OPTS
000027         RETURN
         C--F-P-D-MCC MOVE
000030   30    CALL GRADS(2)
         C--MN IS NO. OF MOVES FOR THIS VALUE OF RHO
000032         IF (MN.NE.0) GO TO 70
000033   40    IREP=0
000034         IT=0
         C--SET INITIAL GUESS INVERS MATRIX OF SECOND PARTIAL DERIVATIVES
         C-- USE PARTIAL INVERSE IF KNOWN
000035         DO 50 I=1,N
000036         DO 50 J=1,N
000037   50    A(I,J)=0.0
000050         DO 60 I=1,N
000051   60    A(I,I)=1.0
000060   70    DO 80 I=1,N
000062   80    DELX(I)=DELX0(I)
000066         IF (IREP.GT.N) GO TO 40
000071         IF (IT.EQ.0) GO TO 130
000072         DO 90 I=1,N
000073         SIG(I)=X(I)-XXX(I)
000076   90    YY(I)=DELL(I)-DELX0(I)
         C--NEGATIVE GRADIENT STORED  AND COMPUTED
         C--COMPUTE  HY
000102         DO 100 I=1,N
```

```
000104          DELX(I)=0.0                                            015180
000105          DO 100 J=1,N                                           015190
000107    100   DELX(I)=DELX(I)+A(I,J)*YY(J)                           015200
          C--COMPUTE  Y(SIG -HY)-1                                     015210
000122          ZCON=0.0                                               015220
000123          DO 110 I=1,N                                           015230
000124    110   ZCON=ZCON+YY(I)*(SIG(I)-DELX(I))                       015240
000132          IF (ZCON.EQ.0.0) GO TO 130                             015250
000133          IREP=IREP+1                                            015260
000135          ZC=1./ZCON                                             015270
          C-- UPDATE   H MATRIX USING  MCC FORMULA WHEN SCALAR NE 0    015280
000136          DO 120 I=1,N                                           015290
000140          T1=ZC*(SIG(I)-DELX(I))                                 015300
000143          DO 120 J=1,N                                           015310
000144          A(I,J)=A(I,J)+T1*(-DELX(J)+SIG(J))                     015320
000153    120   A(J,I)=A(I,J)                                          015330
          C-- STORE CURRENT POINT AND CURRENT GRADIENT (NEG)           015340
000165    130   DO 140 I=1,N                                           015350
000167          XXX(I)=X(I)                                            015360
000171    140   DELL(I)=DELX0(I)                                       015370
000174          DO 150 I=1,N                                           015380
000176          DELX(I)=0.0                                            015390
000177          DO 150 J=1,N                                           015400
000201    150   DELX(I)=DELX(I)+A(I,J)*DELX0(J)                        015410
000214          ZC1=0.0                                                015420
000215          DO 160 I=1,N                                           015430
000216    160   ZC1=DELX(I)**2+ZC1                                     015440
000223          ZC1=SQRT(ZC1)                                          015450
000225          DO 170 I=1,N                                           015460
000226    170   DELX(I)=DELX(I)/ZC1                                    015470
000232          CALL STORES
000233          CALL OPTS
000234          IT=IT+1                                                015500
000236          RETURN                                                 015510
000236    180   CONTINUE                                               015520
          C STEEPEST DESCENT                                           015530
000236          CALL GRADS(2)
000240          DO 190 I=1,N                                           015550
000242    190   DELX(I)=DELX0(I)                                       015560
000246          CALL STORES
000247          CALL OPTS
000250          RETURN                                                 015590
000251          END                                                    015600
```

User-Supplied Subroutines for Inside Programs

```
                          SUBROUTINE READIX
000002           9999     CONTINUE
000002                    RETURN
000003                    END
```

```
                SUBROUTINE RESTNX(IN,VAL)
000005          COMMON/PROB/ISP
000005          COMMON/IN/W(100)
000005          COMMON/SHARES/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
000005          IF(ISP-1)1000,1000,2000
000007    1000  FN=N
000011          VAL=0.
000011          IF(IN)1100,1100,1200
000013    1100  DO 1150 J=1,N
000015    1150  VAL=VAL+W(J)*(X(J)-2.)**2
000023          GO TO 9999
000024    1200  VAL=FN
000025          DO 1250 J=1,N
000027    1250  VAL=VAL-X(J)
000033          GO TO 9999
000033    2000  FN=N
000035          VAL=0.
000035          IF(IN)2100,2100,2200
000037    2100  DO 2150 J=1,N
000041    2150  VAL = VAL + W(J)**.5 * (X(J)-2.)**2
000053          GO TO 9999
000054    2200  VAL=FN
000055          DO 2250 J=1,N
000057    2250  VAL=VAL-X(J)**2
000063          GO TO 9999
000064    9999  CONTINUE
000064          RETURN
000065          END
```

```
                SUBROUTINE GRADIS(IN)
000003          COMMON/PROB/ISP
000003          COMMON/IN/W(100)
000003          COMMON/SHARES/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
000003          DO 500 J=1,N
000005     500  DEL(J)=0.
000010          IF(ISP-1)1000,1000,2000
000013    1000  IF(IN)1100,1100,1200
000015    1100  DO 1150 J=1,N
000017    1150  DEL(J) = 2.*W(J)*(X(J)-2.)
000025          GO TO 9999
000026    1200  DO 1250 J=1,N
000030    1250  DEL(J)=-1.
000034          GO TO 9999
000034    2000  IF(IN)2100,2100,2200
000036    2100  DO 2150 J=1,N
000040    2150  DEL(J) = 2. * W(J)**.5 * (X(J)-2.)
000052          GO TO 9999
000053    2200  DO 2250 J=1,N
000055    2250  DEL(J) =-2.*X(J)
000061          GO TO 9999
000062    9999  CONTINUE
000062          RETURN
000063          END
```

```
                SUBROUTINE MATRXX (IN,L)
000005          COMMON/PROB/ISP
000005          COMMON/IN/W(100)
000005          COMMON/SHARES/X(100),DEL(100),A(100,100),N,M,MN,NP1,NM1
000005          IF(ISP-1)1000,1000,2000
000007     1000 IF(IN)1100,1100,1200
000011     1100 DO 1150 J=1,N
000013     1150 A(J,J) = 2.*W(J)
000023          GO TO 9999
000023     1200 CONTINUE
000023          GO TO 9999
000024     2000 IF(IN)2100,2100,2200
000026     2100 DO 2150 J=1,N
000030     2150 A(J,J)= 2.*W(J)**.5
000044          GO TO 9999
000045     2200 DO 2250 J=1,N
000047     2250 A(J,J)=-2.
000056          GO TO 9999
000056     9999 CONTINUE
000056          RETURN
000057          END
```

USER-SUPPLIED INFORMATION CARDS AND RESULTS PRINTING OUT ALL POINTS OF INSIDE PROGRAMS

It is useful in debugging the overall program and in increasing solution efficiency to print out the solutions of the inside programs, preferably all points. The starting points of the inside programs should be feasible, which is dependent upon the starting point of the outside program. The starting point of the outside program should also be feasible. The solutions of the inside program for the starting point of the outside program should be examined. Also, several points of the outside program, and the corresponding inside program solutions, should be examined.

The SUB computer program, leaving in the statements for printing of points of the inside program, provides the opportunity to accomplish the above, when used with the user-supplied information cards given here.

User-supplied information cards are: parameter card, initial starting point, option card, tolerance card and second option card. Three sets of user-supplied information cards are necessary for the example problem.

The outside program has, on the parameter card, information that EPSI = 1.E-05, RHOIN = 10., THETAO = 1.E-05, RATIO = 10., TMMAX = 360., M = 2, N = 4, MZ = 0. The initial starting point is $x_1 = x_2 = x_3 = x_4 = 4$. The option card indicates that all points of the outside program are to be printed out (Option 3 = 2), and that there is at least one nonlinear constraint (Option 10 = 1). The tolerance card indicates that for numerical differentiation and control of P-function minimization the tolerances are .001. Finally, the second option card indicates that the problem is to be solved without checking first derivatives (Option 1 = 1), and that steepest descent is used to minimize the P-function (Option 2 = 3).

The inside program information cards are similar, with the following exceptions. Each problem has one constraint (M = 1). The starting points are $v_1 = v_2 = v_3 = v_4 = .5$. The first option cards indicate that all subproblem solutions of the inside programs are to be printed out (Option 3 = 1). The first option cards also indicate that the first inside program has all linear constraints (Option 10 = 2) and the second inside program has at least one non-linear constraint (Option 10 = 1). The second option cards for both inside programs indicate that the second-order method, called generalized Newton-Raphson, is used to minimize the P-function (Option 2 = 1).

The printout of results gives initial information for the outside program.

It indicates that Problem A is being solved. It gives initial information for Problem A, noting that SUB is used. It solves Problem A for the initial starting point of the outside program, requiring 16 points, and indicates that Problem A is solved.

It indicates that Problem B is being solved. It gives initial information for Problem B, noting that SUB is used. It solves Problem B for the initial starting point of the outside program, requiring 16 points, and indicates that Problem B is solved.

It prints out information for the outside program, namely the starting point values of the constraints found by solving Problem A and Problem B, and the feasible starting point to be used in the overall procedure.

It then prints that Problem A is being solved. The initial information is not printed this time, however, it having been by-passed in SUB after the first time. Again, Problem A is solved. The printout is given up to point 4 of this solution. The remainder, not included here, is similar for more points of the outside program.

User-Supplied Information Cards

```
1.E-05        10.      1.E-05       10.        360.   2   4   0
    4.         4.          4.        4.
3       1     2      1     1     1     1     1     1     1
  .001        .001
1       3
1.E-05        100.     1.E-05       10.        180.   1   4   0
    .5         .5         .5         .5
3       1     1      1     1     1     1     1     1     2
  .001        .001
1       1
1.E-05        100.     1.E-05       10.        180.   1   4   0
    .5         .5         .5         .5
3       1     1      1     1     1     1     1     1     1
  .001        .001
1       1
```

Printout of Results

```
NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4  3/22/71
    N=  4      M=  2      MZ=  0
      MAX. TIME= 3.6000000E+02    R= 1.0000000E+01    RATIO= 1.0000000E+01      EPSILON= 1.0000000E-05    THETA= 1.0000000E-05
OPTIONS SELECTED
    3      1      2      1      1      1      1      1      1      1
TOLERANCES
     1.0000000E-03       1.0000000E-03
SECOND SET OF OPTIONS
    1      3
    TIME=      .025 SECONDS
```

PROBLEM A

```
NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4    SUB

    N=  4      M=  1      MZ=  0

      MAX. TIME= 1.8000000E+02    P= 1.0000000E+02    RATIO= 1.0000000E+01     EPSILON= 1.0000000E-05    THETA= 1.0000000E-05

OPTIONS SELECTED
    3      1      1      1      1      1      1      1      1      2

TOLERANCES
     1.0000000E-03       1.0000000E-03

SECOND SET OF OPTIONS
     1     1
    TIME=     0.000 SECONDS
      F=  3.6000000E+01      P=  0.                G=  0.                RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
     5.0000000E-01       5.0000000E-01       5.0000000E-01       5.0000000E-01
    THE CONSTRAINT VALUES
                               NOT INCLUDING THE NON-NEGATIVITIES
     2.0000000E+00
    TIME=     .014 SECONDS
*****THE FEASIBLE STARTING POINT TO BE USED IS ...
      F=  3.6000000E+01      P=  0.                G=  0.                RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
     5.0000000E-01       5.0000000E-01       5.0000000E-01       5.0000000E-01
    THE CONSTRAINT VALUES
                               NOT INCLUDING THE NON-NEGATIVITIES
     2.0000000E+00
    TIME=     .073 SECONDS
*******************************************
        POINT=    2        DOTT=  6.4890716E-07      RHO=  1.0000000E+02      MAGNITUDE=  2.3681602E-02      PHASE= 2
      F=  2.2595486E+01      P=  1.3438514E+02     G= -4.7740451E+02     RSIGMA=  1.1178965E+02     H=  0.
    THE CURRENT VALUE OF X IS
     8.1163225E-01       8.1163225E-01       8.1163225E-01       8.1163225E-01
    THE CONSTRAINT VALUES
                               NOT INCLUDING THE NON-NEGATIVITIES
     7.5347099E-01


  LAGRANGE MULTIPLIERS
      F=  2.2595486E+01      P=  1.3438514E+02     G= -4.7740451E+02     RSIGMA=  1.1178965E+02     H=  0.
    THE CURRENT VALUE OF X IS
     1.2320851E+02       1.2320851E+02       1.2320851E+02       1.2320851E+02
    THE CONSTRAINT VALUES
                               NOT INCLUDING THE NON-NEGATIVITIES
     1.3271911E+02
    TIME=     .145 SECONDS
*******************************************
        POINT=    4        DOTT=  1.3778905E-07      RHO=  1.0000000E+01      MAGNITUDE=  5.0772586E-03      PHASE= 2
      F=  2.0166114E+01      P=  3.2520721E+01     G= -2.9833886E+01     RSIGMA=  1.2354607E+01     H=  0.
    THE CURRENT VALUE OF X IS
     8.7733258E-01       8.7733258E-01       8.7733258E-01       8.7733258E-01
    THE CONSTRAINT VALUES
                               NOT INCLUDING THE NON-NEGATIVITIES
     4.9066966E-01


   1ST  ORDER ESTIMATES
      F=  1.9904710E+01      P=  2.1202452E+01     G=  1.9896184E+01     RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
```

```
 8.8463262E-01        8.8463262E-01        8.8463262E-01        8.8463262E-01
THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
 4.6146952E-01


LAGRANGE MULTIPLIERS
     F=  1.9904710E+01       P=  2.1202452E+01       G=  1.9896184E+01       RSIGMA=  0.                 H=  0.
   THE CURRENT VALUE OF X IS
    1.1398186E+01        1.1398186E+01        1.1398186E+01        1.1398186E+01
   THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
    2.0380310E+01
   TIME=     .233 SECONDS
*******************************************
       POINT=   6        DOTT=  3.6201352E-07       RHO=  1.0000000E+00       MAGNITUDE=  1.1263842E-02       PHASE= 2
     F=  1.6876951E+01       P=  1.9210771E+01       G=  1.1876951E+01       RSIGMA=  2.3338206E+00      H=  0.
   THE CURRENT VALUE OF X IS
    9.7296084E-01        9.7296084E-01        9.7296084E-01        9.7296084E-01
   THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
    1.0815663E-01


  2ND  ORDER ESTIMATES
     F=  1.6497058E+01       P=  1.7696832E+01       G=  1.6511488E+01       RSIGMA=  0.                 H=  0.
   THE CURRENT VALUE OF X IS
    9.8458573E-01        9.8458573E-01        9.8458573E-01        9.8458573E-01
   THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
    6.1657062E-02


  1ST  ORDER ESTIMATES
     F=  1.6529552E+01       P=  1.7731888E+01       G=  1.6511488E+01       RSIGMA=  0.                 H=  0.
   THE CURRENT VALUE OF X IS
    9.8358620E-01        9.8358620E-01        9.8358620E-01        9.8358620E-01
   THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
    6.5655187E-02


LAGRANGE MULTIPLIERS
     F=  1.6529552E+01       P=  1.7731888E+01       G=  1.6511488E+01       RSIGMA=  0.                 H=  0.
   THE CURRENT VALUE OF X IS
    1.0277906E+00        1.0277906E+00        1.0277906E+00        1.0277906E+00
   THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
    9.2458498E+00
   TIME=     .335 SECONDS
*******************************************
       POINT=   8        DOTT=  5.5412877E-08       RHO=  1.0000000E-01       MAGNITUDE=  1.2123883E-02       PHASE= 2
     F=  1.6098637E+01       P=  1.6539599E+01       G=  1.5598637E+01       RSIGMA=  4.4096222E-01      H=  0.
   THE CURRENT VALUE OF X IS
    9.9692234E-01        9.9692234E-01        9.9692234E-01        9.9692234E-01
   THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
    1.2310625E-02


  2ND  ORDER ESTIMATES
     F=  1.6008118E+01       P=  1.6227761E+01       G=  1.6012157E+01       RSIGMA=  0.                 H=  0.
   THE CURRENT VALUE OF X IS
    9.9974633E-01        9.9974633E-01        9.9974633E-01        9.9974633E-01
```

```
THE CONSTRAINT VALUES
                         NOT INCLUDING THE NON-NEGATIVITIES
 1.0146632E-03

1ST  ORDER ESTIMATES
  F=  1.6013291E+01     P=  1.6242802E+01     G=  1.6012157E+01     RSIGMA=  0.                  H=  0.
THE CURRENT VALUE OF X IS
 9.9958473E-01      9.9958473E-01      9.9958473E-01      9.9958473E-01
THE CONSTRAINT VALUES
                         NOT INCLUDING THE NON-NEGATIVITIES
 1.6610684E-03

LAGRANGE MULTIPLIERS
  F=  1.6013291E+01     P=  1.6242802E+01     G=  1.6012157E+01     RSIGMA=  0.                  H=  0.
THE CURRENT VALUE OF X IS
 1.0030872E-01      1.0030872E-01      1.0030872E-01      1.0030872E-01
THE CONSTRAINT VALUES
                         NOT INCLUDING THE NON-NEGATIVITIES
 8.1230644E+00
TIME=     .433 SECONDS
*******************************************
     POINT=  10      DOTT=  5.8714668E-09     RHO=  1.0000000E-02     MAGNITUDE=  1.2290563E-02     PHASE= 2
  F=  1.6009988E+01     P=  1.6076860E+01     G=  1.5959988E+01     RSIGMA=  6.6871855E-02     H=  0.
THE CURRENT VALUE OF X IS
 9.9968791E-01      9.9968791E-01      9.9968791E-01      9.9968791E-01
THE CONSTRAINT VALUES
                         NOT INCLUDING THE NON-NEGATIVITIES
 1.2483446E-03

2ND  ORDER ESTIMATES
  F=  1.6000021E+01     P=  1.6023249E+01     G=  1.6000139E+01     RSIGMA=  0.                  H=  0.
THE CURRENT VALUE OF X IS
 9.9999935E-01      9.9999935E-01      9.9999935E-01      9.9999935E-01
THE CONSTRAINT VALUES
                         NOT INCLUDING THE NON-NEGATIVITIES
 2.6177795E-06

1ST  ORDER ESTIMATES
  F=  1.6000154E+01     P=  1.6025445E+01     G=  1.6000139E+01     RSIGMA=  0.                  H=  0.
THE CURRENT VALUE OF X IS
 9.9999520E-01      9.9999520E-01      9.9999520E-01      9.9999520E-01
THE CONSTRAINT VALUES
                         NOT INCLUDING THE NON-NEGATIVITIES
 1.9202286E-05

LAGRANGE MULTIPLIERS
  F=  1.6000154E+01     P=  1.6025445E+01     G=  1.6000139E+01     RSIGMA=  0.                  H=  0.
THE CURRENT VALUE OF X IS
 1.0003122E-02      1.0003122E-02      1.0003122E-02      1.0003122E-02
THE CONSTRAINT VALUES
                         NOT INCLUDING THE NON-NEGATIVITIES
 8.0106089E+00
TIME=     .527 SECONDS
*******************************************
     POINT=  12      DOTT=  1.9072340E-08     RHO=  1.0000000E-03     MAGNITUDE=  7.0193568E-02     PHASE= 2
  F=  1.6001001E+01     P=  1.6009987E+01     G=  1.5996001E+01     RSIGMA=  8.9861545E-03     H=  0.
THE CURRENT VALUE OF X IS
 9.9996871E-01      9.9996871E-01      9.9996871E-01      9.9996871E-01
THE CONSTRAINT VALUES
```

```
                        NOT INCLUDING THE NON-NEGATIVITIES
  1.2514601E-04

  2ND  ORDER ESTIMATES
     F= 1.6000001E+01     P= 1.6002326E+01     G= 1.6000003E+01     RSIGMA= 0.              H= 0.
    THE CURRENT VALUE OF X IS
     9.9999996E-01      9.9999996E-01      9.9999996E-01      9.9999996E-01
    THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
  1.5571086E-07

  1ST  ORDER ESTIMATES
     F= 1.6000003E+01     P= 1.6002557E+01     G= 1.6000003E+01     RSIGMA= 0.              H= 0.
    THE CURRENT VALUE OF X IS
     9.9999991E-01      9.9999991E-01      9.9999991E-01      9.9999991E-01
    THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
  3.4617658E-07

 LAGRANGE MULTIPLIERS
     F= 1.6000003E+01     P= 1.6002557E+01     G= 1.6000003E+01     RSIGMA= 0.              H= 0.
    THE CURRENT VALUE OF X IS
     1.0000313E-03      1.0000313E-03      1.0000313E-03      1.0000313E-03
    THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
  7.9906660E+00
    TIME=    .618 SECONDS
*******************************************
       POINT= 14        DOTT= 3.4340067E-07     RHO= 1.0000000E-04     MAGNITUDE= 8.8571925E-01     PHASE= 2
     F= 1.6000098E+01     P= 1.6001229E+01     G= 1.5999598E+01     RSIGMA= 1.1312673E-03     H= 0.
    THE CURRENT VALUE OF X IS
     9.9999695E-01      9.9999695E-01      9.9999695E-01      9.9999695E-01
    THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
  1.2217264E-05

  2ND  ORDER ESTIMATES
     F= 1.5999997E+01     P= 1.6000233E+01     G= 1.5999997E+01     RSIGMA= 0.              H= 0.
    THE CURRENT VALUE OF X IS
     1.0000001E+00      1.0000001E+00      1.0000001E+00      1.0000001E+00
    THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
 -3.3720913E-07

  1ST  ORDER ESTIMATES
     F= 1.5999997E+01     P= 1.6000256E+01     G= 1.5999997E+01     RSIGMA= 0.              H= 0.
    THE CURRENT VALUE OF X IS
     1.0000001E+00      1.0000001E+00      1.0000001E+00      1.0000001E+00
    THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
 -3.3037527E-07

 LAGRANGE MULTIPLIERS
     F= 1.5999997E+01     P= 1.6000256E+01     G= 1.5999997E+01     RSIGMA= 0.              H= 0.
    THE CURRENT VALUE OF X IS
     1.0000031E-04      1.0000031E-04      1.0000031E-04      1.0000031E-04
    THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
  8.1851389E+00
```

```
TIME=     .706 SECONDS

*******************************************
        POINT=  16        DOTT=  1.6627993E-07      RHO=  1.0000000E-05      MAGNITUDE=  1.8275368E+00       PHASE= 2
       F=  1.6000009E+01      P=  1.6000146E+01      G=  1.5999959E+01      RSIGMA=  1.3655964E-04      H=  0.
    THE CURRENT VALUE OF X IS
     9.9999971E-01       9.9999971E-01       9.9999971E-01       9.9999971E-01
    THE CONSTRAINT VALUES
                               NOT INCLUDING THE NON-NEGATIVITIES
     1.1729808E-06

   2ND  ORDER ESTIMATES
       F=  1.6000000E+01      P=  1.6000023E+01      G=  1.6000000E+01      RSIGMA=  0.                 H=  0.
    THE CURRENT VALUE OF X IS
     1.0000000E+00       1.0000000E+00       1.0000000E+00       1.0000000E+00
    THE CONSTRAINT VALUES
                               NOT INCLUDING THE NON-NEGATIVITIES
    -5.1371686E-08

   1ST  ORDER ESTIMATES
       F=  1.6000000E+01      P=  1.6000026E+01      G=  1.6000000E+01      RSIGMA=  0.                 H=  0.
    THE CURRENT VALUE OF X IS
     1.0000000E+00       1.0000000E+00       1.0000000E+00       1.0000000E+00
    THE CONSTRAINT VALUES
                               NOT INCLUDING THE NON-NEGATIVITIES
    -5.4161688E-08

  LAGRANGE MULTIPLIERS
       F=  1.6000000E+01      P=  1.6000026E+01      G=  1.6000000E+01      RSIGMA=  0.                 H=  0.
    THE CURRENT VALUE OF X IS
     1.0000003E-05       1.0000003E-05       1.0000003E-05       1.0000003E-05
    THE CONSTRAINT VALUES
                               NOT INCLUDING THE NON-NEGATIVITIES
     8.5252888E+00
```

PROBLEM A SOLVED

PROBLEM H

```
NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4    SUR

    N=  4      M=  1      MZ=  0

      MAX. TIME= 1.8000000E+02    R= 1.0000000E+02    RATIO= 1.0000000E+01      EPSILON= 1.0000000F-05    THETA= 1.0000000E-05

OPTIONS SELECTED
    3      1      1      1      1      1      1      1      1      1

TOLERANCES
     1.0000000E-03       1.0000000E-03

SECOND SET OF OPTIONS
    1      1
    TIME=    0.000 SECONDS
      F=  1.8000000E+01      P=  0.                 G=  0.                RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
     5.0000000E-01       5.0000000E-01       5.0000000E-01       5.0000000E-01
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     3.0000000E+00
    TIME=     .014 SECONDS

*****THE FEASIBLE STARTING POINT TO BE USED IS ...
      F=  1.8000000E+01      P=  0.                 G=  0.                RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
     5.0000000E-01       5.0000000E-01       5.0000000E-01       5.0000000E-01
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     3.0000000E+00
    TIME=     .112 SECONDS

*******************************************
        POINT=   2       DOTT=  6.6344653E-07      RHO=  1.0000000E+02      MAGNITUDE=  2.5042075E-02      PHASE= 2
      F=  1.1108596E+01      P=  6.3481406E+01     G= -4.8889140E+02     RSIGMA=  5.2372810E+01     H=  0.
    THE CURRENT VALUE OF X IS
     8.2162208E-01       8.2162208E-01       8.2162208E-01       8.2162208E-01
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     1.2997486E+00


 LAGRANGE MULTIPLIERS
      F=  1.1108596E+01      P=  6.3481406E+01     G= -4.8889140E+02     RSIGMA=  5.2372810E+01     H=  0.
    THE CURRENT VALUE OF X IS
     1.2171046E+02       1.2171046E+02       1.2171046E+02       1.2171046E+02
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     7.6937953E+01
    TIME=     .217 SECONDS

*******************************************
        POINT=   4       DOTT=  7.4900159E-08      RHO=  1.0000000E+01      MAGNITUDE=  3.2592589E-03      PHASE= 2
      F=  1.0438018E+01      P=  1.6020672E+01     G= -3.9561982E+01     RSIGMA=  5.5826542E+00     H=  0.
    THE CURRENT VALUE OF X IS
     8.5774248E-01       8.5774248E-01       8.5774248E-01       8.5774248E-01
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     1.0571113E+00


   1ST  ORDER ESTIMATES
      F=  1.0364798E+01      P=  1.0747257E+01     G=  1.0363509E+01     RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
```

```
  8.6175586E-01       9.6175586E-01       8.6175586E-01       8.6175586E-01
 THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
  1.0295073E+00


LAGRANGE MULTIPLIERS
    F=  1.0364798E+01     P=  1.0747257E+01     G=  1.0363509E+01     RSIGMA=  0.               H=  0.
 THE CURRENT VALUE OF X IS
  1.1658511E+01       1.1658511E+01       1.1658511E+01       1.1658511E+01
 THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
  9.4597417E+00
 TIME=     .341 SECONDS
*******************************************
       POINT=   6        DOTT=  3.4565497E-10     RHO=  1.0000000E+00     MAGNITUDE=  2.0385174E-04      PHASE= 2
    F=  8.7631835E+00     P=  9.9641443E+00     G=  3.7631835E+00     RSIGMA=  1.2009608E+00     H=  0.
 THE CURRENT VALUE OF X IS
  9.5338740E-01       9.5338740E-01       9.5338740E-01       9.5338740E-01
 THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
  3.6420989E-01


  2ND  ORDER ESTIMATES
    F=  8.5690130E+00     P=  9.2764891E+00     G=  8.5770908E+00     RSIGMA=  0.               H=  0.
 THE CURRENT VALUE OF X IS
  9.6504753E-01       9.6504753E-01       9.6504753E-01       9.6504753E-01
 THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
  2.7473309E-01


  1ST  ORDER ESTIMATES
    F=  8.5861258E+00     P=  9.2911968E+00     G=  8.5770908E+00     RSIGMA=  0.               H=  0.
 THE CURRENT VALUE OF X IS
  9.6401461E-01       9.6401461E-01       9.6401461E-01       9.6401461E-01
 THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
  2.8270333E-01


LAGRANGE MULTIPLIERS
    F=  8.5861258E+00     P=  9.2911968E+00     G=  8.5770908E+00     RSIGMA=  0.               H=  0.
 THE CURRENT VALUE OF X IS
  1.0488916E+00       1.0488916E+00       1.0488916E+00       1.0488916E+00
 THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
  2.7456695E+00
 TIME=     .480 SECONDS
*******************************************
       POINT=   8        DOTT=  1.9350286E-08     RHO=  1.0000000E-01     MAGNITUDE=  3.6524186E-03      PHASE= 2
    F=  8.0969863E+00     P=  8.4026486E+00     G=  7.5969863E+00     RSIGMA=  3.0566231E-01     H=  0.
 THE CURRENT VALUE OF X IS
  9.9395662E-01       9.9395662E-01       9.9395662E-01       9.9395662E-01
 THE CONSTRAINT VALUES
                       NOT INCLUDING THE NON-NEGATIVITIES
  4.8200967E-02


  2ND  ORDER ESTIMATES
    F=  8.0190147E+00     P=  8.2184213E+00     G=  8.0229644E+00     RSIGMA=  0.               H=  0.
 THE CURRENT VALUE OF X IS
  9.9881229E-01       9.9881229E-01       9.9881229E-01       9.9881229E-01
```

```
THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
 9.4960700E-03


IST  ORDER ESTIMATES
   F=  8.0245899E+00     P=  8.2291491E+00     G=  8.0229644E+00     RSIGMA=  0.                H=  0.
 THE CURRENT VALUE OF X IS
  9.9846431E-01       9.9846431E-01       9.9846431E-01       9.9846431E-01
 THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
 1.2276093E-02


LAGRANGE MULTIPLIERS
   F=  8.0245899E+00     P=  8.2291491E+00     G=  8.0229644E+00     RSIGMA=  0.                H=  0.
 THE CURRENT VALUE OF X IS
  1.0060801E-01       1.0060801E-01       1.0060801E-01       1.0060801E-01
 THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
 2.0746472E+00
 TIME=     .609 SECONDS
*****************************************
      POINT=  10      OOTT=  8.6014629E-09     RHO=  1.0000000E-02     MAGNITUOE=  7.4404491E-03     PHASE= 2
   F=  8.0099660E+00     P=  8.0630144E+00     G=  7.9599660E+00     RSIGMA=  5.3048390E-02     H=  0.
 THE CURRENT VALUE OF X IS
  9.9937732E-01       9.9937732E-01       9.9937732E-01       9.9937732E-01
 THE CONSTRAINT VALUES
                        NOT INCLUOING THE NON-NEGATIVITIFS
 4.9798896E-03


2ND  ORDER ESTIMATES
   F=  8.0000812E+00     P=  8.0232179E+00     G=  8.0002971E+00     RSIGMA=  0.                H=  0.
 THE CURRENT VALUE OF X IS
  9.9999493E-01       9.9999493E-01       9.9999493E-01       9.9999493E-01
 THE CONSTRAINT VALUES
                        NOT INCLUOING THE NON-NEGATIVITIES
 4.0589271E-05


IST  ORDER ESTIMATES
   F=  8.0003261E+00     P=  8.0252772E+00     G=  8.0002971E+00     RSIGMA=  0.                H=  0.
 THE CURRENT VALUE OF X IS
  9.9997962E-01       9.9997962E-01       9.9997962E-01       9.9997962E-01
 THE CONSTRAINT VALUES
                        NOT INCLUOING THE NON-NEGATIVITIES
 1.6303709E-04


LAGRANGE MULTIPLIERS
   F=  8.0003261E+00     P=  8.0252772E+00     G=  8.0002971E+00     RSIGMA=  0.                H=  0.
 THE CURRENT VALUE OF X IS
  1.0006231E-02       1.0006231E-02       1.0006231E-02       1.0006231E-02
 THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
 2.0080766E+00
 TIME=     .738 SECONOS
*****************************************
      POINT=  12      OOTT=  5.6830703E-08     RHO=  1.0000000E-03     MAGNITUOE=  5.9880157E-02     PHASE= 2
   F=  8.0009973E+00     P=  8.0086012E+00     G=  7.9959973E+00     RSIGMA=  7.6039334E-03     H=  0.
 THE CURRENT VALUE OF X IS
  9.9993767E-01       9.9993767E-01       9.9993767E-01       9.9993767E-01
 THE CONSTRAINT VALUES
```

```
                     NOT INCLUDING THE NON-NEGATIVITIES
  4.9861113E-04

 2ND  ORDER ESTIMATES
    F=  7.9999978E+00     P=  8.0023258E+00     G=  8.0000008E+00     RSIGMA=  0.                H=  0.
  THE CURRENT VALUE OF X IS
   1.0000001E+00       1.0000001E+00       1.0000001E+00       1.0000001E+00
  THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
  -1.1052083E-06

 1ST  ORDER ESTIMATES
    F=  8.0000011E+00     P=  8.0025553E+00     G=  8.0000008E+00     RSIGMA=  0.                H=  0.
  THE CURRENT VALUE OF X IS
   9.9999993E-01       9.9999993E-01       9.9999993E-01       9.9999993E-01
  THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
   5.3623128E-07

LAGRANGE MULTIPLIERS
    F=  8.0000011E+00     P=  8.0025553E+00     G=  8.0000008E+00     RSIGMA=  0.                H=  0.
  THE CURRENT VALUE OF X IS
   1.0000623E-03       1.0000623E-03       1.0000623E-03       1.0000623E-03
  THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
   2.0055710E+00
  TIME=     .856 SECONDS
*********************************************
    POINT=  14       DDTT=  1.8815766E-07      RHO=  1.0000000E-04      MAGNITUDE=  3.6276577E-01      PHASE= 2
    F=  8.0010011E+00     P=  8.0010904E+00     G=  7.9996011E+00     RSIGMA=  9.8926685E-04     H=  0.
  THE CURRENT VALUE OF X IS
   9.9999368E-01       9.9999368E-01       9.9999368E-01       9.9999368E-01
  THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
   5.0545165E-05

 2ND  ORDER ESTIMATES
    F=  8.0000015E+00     P=  8.0002326E+00     G=  8.0000015E+00     RSIGMA=  0.                H=  0.
  THE CURRENT VALUE OF X IS
   9.9999990E-01       9.9999990E-01       9.9999990E-01       9.9999990E-01
  THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
   7.6075462E-07

 1ST  ORDER ESTIMATES
    F=  8.0000015E+00     P=  8.0002558E+00     G=  8.0000015E+00     RSIGMA=  0.                H=  0.
  THE CURRENT VALUE OF X IS
   9.9999991E-01       9.9999991E-01       9.9999991E-01       9.9999991E-01
  THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
   7.5850938E-07

LAGRANGE MULTIPLIERS
    F=  8.0000015E+00     P=  8.0002558E+00     G=  8.0000015E+00     RSIGMA=  0.                H=  0.
  THE CURRENT VALUE OF X IS
   1.0000063E-04       1.0000063E-04       1.0000063E-04       1.0000063E-04
  THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
   1.9784286E+00
```

```
TIME=     .969 SECONDS

..............................................
        POINT=  16       DOTT=  4.0246536E-0T      RHO=  1.0000000E-05      MAGNITUDE=  1.336755TE+00      PHASE= 2
      F=  8.0000089E+00      P=  8.000132IE+00      G=  7.9999589E+00      RSIGMA=  I.232T88IF-04      H=  0.
    THE CURRENT VALUE OF X IS
     9.9999945E-0I       7.9999945E-01       9.9999945E-01       9.9999945E-0I
    THE CONSTRAINT VALUES
                         NOT INCLUDING THE NON-NEGATIVITIES
     4.4265993E-06

   2ND  ORDER ESTIMATES
      F=  T.9999986E+00      P=  8.0000233E+00      G=  T.9999986E+00      RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
     1.0000001E+00       1.0000001E+00       I.000000IE+00       1.000000IE+00
    THE CONSTRAINT VALUES
                         NOT INCLUDING THE NON-NEGATIVITIFS
    -T.I241I36E-07

   IST  ORDER ESTIMATES
      F=  T.9999986E+00      P=  8.000025TE+00      G=  T.9999986E+00      RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
     I.0000001E+00       1.000000IE+00       1.0000001E+00       I.000000IE+00
    THE CONSTRAINT VALUES
                         NOT INCLUDING THE NON-NEGATIVITIFS
    -6.9770213E-0T

  LAGRANGE MULTIPLIERS
      F=  7.9999986E+00      P=  8.000025TE+00      G=  T.9999986E+00      RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
     1.0000006E-05       I.0000006E-05       1.0000006E-05       1.0000006E-05
    THE CONSTRAINT VALUES
                         NOT INCLUDING THE NON-NEGATIVITIES
     2.2590705E+00
```

```
PROBLEM 8 SOLVED
     F=  1.6000000E+01      P=  0.                G=  0.                RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
     4.0000000E+00       4.0000000E+00       4.0000000E+00       4.0000000E+00
    THE CONSTRAINT VALUES
                            NOT INCLUDING THE NON-NEGATIVITIES
     1.2000009E+01       4.0000089E+00
    TIME=    1.900 SECONDS
*****THE FEASIBLE STARTING POINT TO BE USED IS ...
     F=  1.6000000E+01      P=  0.                G=  0.                RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
     4.0000000E+00       4.0000000E+00       4.0000000E+00       4.0000000E+00
    THE CONSTRAINT VALUES
                            NOT INCLUDING THE NON-NEGATIVITIES
     1.2000009E+01       4.0000089E+00
```

```
PROBLEM A
    TIME=    0.000 SECONDS
      F=  3.6000000E+01      P=  0.                G=  0.               RSIGMA=  0.              H=  0.
    THE CURRENT VALUE OF X IS
     5.0000000E-01      5.0000000E-01      5.0000000E-01      5.0000000E-01
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     2.0000000E+00
    TIME=     .013 SECONDS

*****THE FEASIBLE STARTING POINT TO BE USED IS ...
      F=  3.6000000E+01      P=  0.                G=  0.               RSIGMA=  0.              H=  0.
    THE CURRENT VALUE OF X IS
     5.0000000E-01      5.0000000E-01      5.0000000E-01      5.0000000E-01
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     2.0000000E+00
    TIME=     .073 SECONDS

********************************************
          POINT=   2        DOTT=  6.4890716E-07      RHO=  1.0000000E+02      MAGNITUDE=  2.3681602E-02      PHASE= 2
      F=  2.2595486E+01      P=  1.3438514E+02     G= -4.7740451E+02     RSIGMA=  1.1178965E+02     H=  0.
    THE CURRENT VALUE OF X IS
     8.1163225E-01      8.1163225E-01      8.1163225E-01      8.1163225E-01
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     7.5347099E-01


  LAGRANGE MULTIPLIERS
      F=  2.2595486E+01      P=  1.3438514E+02     G= -4.7740451E+02     RSIGMA=  1.1178965E+02     H=  0.
    THE CURRENT VALUE OF X IS
     1.2320851E+02      1.2320851E+02      1.2320851E+02      1.2320851E+02
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     1.3271911E+02
    TIME=     .145 SECONDS

********************************************
          POINT=   4        DOTT=  1.3778905E-07      RHO=  1.0000000E+01      MAGNITUDE=  5.0772586E-03      PHASE= 2
      F=  2.0166114E+01      P=  3.2520721E+01     G= -2.9833886E+01     RSIGMA=  1.2354607E+01     H=  0.
    THE CURRENT VALUE OF X IS
     8.7733258E-01      8.7733258E-01      8.7733258E-01      8.7733258E-01
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     4.9066966E-01


   1ST  ORDER ESTIMATES
      F=  1.9904710E+01      P=  2.1202452E+01     G=  1.9896184E+01     RSIGMA=  0.              H=  0.
    THE CURRENT VALUE OF X IS
     8.8463262E-01      8.8463262E-01      8.8463262E-01      8.8463262E-01
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     4.6146952E-01


  LAGRANGE MULTIPLIERS
      F=  1.9904710E+01      P=  2.1202452E+01     G=  1.9896184E+01     RSIGMA=  0.              H=  0.
    THE CURRENT VALUE OF X IS
     1.1398186E+01      1.1398186E+01      1.1398186E+01      1.1398186E+01
    THE CONSTRAINT VALUES
                              NOT INCLUDING THE NON-NEGATIVITIES
     2.0380310E+01
```

RESULTS PRINTING OUT ALL POINTS OF OUTSIDE PROGRAM

The SUMT program is modified by changing RESTNT, and the INSUMT program is modified by changing BODYS, CONVRX, ESTIMS, FEASS, INVERX, OPTS, OUTPUX, PUNCHS, TCHECX, and TIMECS to eliminate printing of inside programs.

The user-supplied information cards are identical to those of the previous section, and are not repeated here. The printouts of the inside programs called for by their information cards are by-passed by the changes in subroutines mentioned above.

The printout of the solution prints initial information for the outside program, for Problem A, and for Problem B. From then on all of the printout pertains to the outside program. At point 17 the value of r has been reduced to 1.E-04, and the solution has been approximately attained. The program cuts off at central processor time equals 360 seconds.

Printout of Results

```
NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4  3/22/71

    N= 4      M= 2      MZ= 0

      MAX. TIME= 3.6000000E+02    R= 1.0000000E+01    RATIO= 1.0000000E+01      EPSILON= 1.0000000E-05    THETA= 1.0000000E-05

OPTIONS SELECTED
    3      1      2      1      1      1      1      1      1      1

TOLERANCES
     1.0000000E-03       1.0000000E-03

SECOND SET OF OPTIONS
    1      3
    TIME=     .027 SECONDS
```

```
NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4     SUB
    N=  4       M=  1       MZ=  0
      MAX. TIME= 1.8000000E+02    R= 1.0000000E+02    RATIO= 1.0000000E+01      EPSILON= 1.0000000E-05    THETA= 1.0000000E-05
OPTIONS SELECTED
    3       1       1       1       1       1       1       1       1       2
TOLERANCES
     1.0000000E-03       1.0000000E-03
SECOND SET OF OPTIONS
    1       1
```

```
NONLINEAR PROGRAMMING ROUTINE-SUMT VERSION 4    SUB

    N=  A       M=  1       47=  0

       MAX. TIME= 1.8000000E+02    R= 1.0000000F+02    RATIO= 1.0000000E+01      EPSILON= 1.0000000F-05    THETA= 1.0000000F-05

OPTIONS SELECTED
    3       1       1       1       1       1       1       1       1       1

TOLERANCES
     1.0000000E-03       1.0000000F-03

SECOND SET OF OPTIONS
    1       1
      F=  1.6000000E+01      P=  0.                  G=  0.                  RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
     A.0000000E+00       4.0000000F+00       4.0000000E+00       4.0000000F+00
    THE CONSTRAINT VALUES
                          NOT INCLUDING THE NON-NEGATIVITIES
     1.2000009E+01       4.0000089F+00
    TIME=     .963 SECONDS

*****THE FEASIBLE STARTING POINT TO BE USED IS ...
      F=  1.6000000E+01      P=  0.                  G=  0.                  RSIGMA=  0.                H=  0.
    THE CURRENT VALUE OF X IS
     A.0000000E+00       4.0000000F+00       4.0000000E+00       4.0000000F+00
    THE CONSTRAINT VALUES
                          NOT INCLUDING THE NON-NEGATIVITIES
     1.2000009E+01       4.0000089F+00
    TIME=    23.600 SECONDS

*******************************************
        POINT=    1         DOTT=  3.5011227E+01      RHO=  1.0000000E+01      MAGNITUDE=  5.9170286E+00       PHASE= 2
      F=  5.7545151E+01      P= -1.1304499E+02      G= -2.4548487E+00      RSIGMA= -1.7059014F+02     H=  0.
    THE CURRENT VALUE OF X IS
     1.4386288E+01       1.4386288F+01       1.4386288E+01       1.4386288E+01
    THE CONSTRAINT VALUES
                          NOT INCLUDING THE NON-NEGATIVITIES
     5.3545259E+01       1.1171714F+01
    TIME=    35.111 SECONDS

*******************************************
        POINT=    2         DOTT=  5.1208486E-08      RHO=  1.0000000E+01      MAGNITUDE=  2.2629292E-04       PHASE= 2
      F=  5.7542415E+01      P= -1.1304499E+02      G= -2.4575846E+00      RSIGMA= -1.7058741F+02     H=  0.
    THE CURRENT VALUE OF X IS
     1.4385604E+01       1.4385604F+01       1.4385604E+01       1.4385604F+01
    THE CONSTRAINT VALUES
                          NOT INCLUDING THE NON-NEGATIVITIES
     5.3542514E+01       1.1171354E+01


  LAGRANGE MULTIPLIERS
      F=  5.7542415F+01      P= -1.1304499E+02      G= -2.4575846E+00      RSIGMA= -1.7058741F+02     H=  0.
    THE CURRENT VALUE OF X IS
     6.9513940E-01       6.9513940F-01       6.9513940E-01       6.9513940F-01
    THE CONSTRAINT VALUES
                          NOT INCLUDING THE NON-NEGATIVITIES
     1.8676747E-01       8.9514667F-01
    TIME=    64.810 SECONDS

*******************************************
        POINT=    3         DOTT=  3.2394380E+00      RHO=  1.0000000E+00      MAGNITUDE=  1.7998439F+00       PHASE= 2
      F=  7.8079487E+00      P=  3.3326411E+00      G=  1.8079487E+00      RSIGMA= -4.4753076F+00     H=  0.
    THE CURRENT VALUE OF X IS
```

```
 1.9519872E+00      1.9519872E+00      1.9519872E+00      1.9519872E+00
THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
 3.8079588E+00      1.5885515E+00
TIME=   80.232 SECONDS
*******************************************
       POINT=   5       DOTT=  2.6754227E-07      RHO=  1.0000000E+00      MAGNITUDE=  5.1724488E-04      PHASE= 2
   F=  7.8084659E+00      P=  3.3326410E+00      G=  1.8084659E+00      RSIGMA= -4.4758249E+00      H=  0.
THE CURRENT VALUE OF X IS
 1.9521165E+00      1.9521165E+00      1.9521165E+00      1.9521165E+00
THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
 3.8084760E+00      1.5887366E+00


1ST  ORDER ESTIMATES
   F=  2.2824715E+00      P=  1.6263489E+01      G=  2.2824715E+00      RSIGMA=  0.              H=  0.
THE CURRENT VALUE OF X IS
 5.7061789E-01      5.7061787E-01      5.7061787E-01      5.7061788E-01
THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
-1.7175275E+00     -9.7842900E-01


LAGRANGE MULTIPLIERS
   F=  2.2824715E+00      P=  1.6263489E+01      G=  2.2824715E+00      RSIGMA=  0.              H=  0.
THE CURRENT VALUE OF X IS
 5.1226451E-01      5.1226452E-01      5.1226452E-01      5.1226452E-01
THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
 2.6257222E-01      6.2043094E-01
TIME=  107.267 SECONDS
*******************************************
       POINT=   6       DOTT=  3.2399114E+00      RHO=  1.0000000E-01      MAGNITUDE=  1.7999754E+00      PHASE= 2
   F=  4.2193837E+00      P=  4.5720697E+00      G=  3.6193837E+00      RSIGMA=  3.5268605E-01      H=  0.
THE CURRENT VALUE OF X IS
 1.0548459E+00      1.0548459E+00      1.0548459E+00      1.0548459E+00
THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
 2.1938451E-01      1.0822856E-01
TIME=  124.671 SECONDS
*******************************************
       POINT=   8       DOTT=  7.6208011E-07      RHO=  1.0000000E-01      MAGNITUDE=  8.7297200E-04      PHASE= 2
   F=  4.2191776E+00      P=  4.5720699E+00      G=  3.6191776E+00      RSIGMA=  3.5289232E-01      H=  0.
THE CURRENT VALUE OF X IS
 1.0547944E+00      1.0547944E+00      1.0547944E+00      1.0547944E+00
THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
 2.1917841E-01      1.0812823E-01


2ND  ORDER ESTIMATES
   F=  3.8359021E+00      P=  4.5930801E+00      G=  3.8203678E+00      RSIGMA=  0.              H=  0.
THE CURRENT VALUE OF X IS
 9.5897553E-01      9.5897552E-01      9.5897552E-01      9.5897552E-01
THE CONSTRAINT VALUES
                     NOT INCLUDING THE NON-NEGATIVITIES
-1.6409711E-01     -8.2907059E-02


1ST  ORDER ESTIMATES
   F=  3.8203678E+00      P=  4.7097842E+00      G=  3.8203678E+00      RSIGMA=  0.              H=  0.
THE CURRENT VALUE OF X IS
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   9.5509196E-01       9.5509194E-01       9.5509194E-01       9.5509194F-01
  THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
  -1.7463133E-01      -9.0846653F-02


LAGRANGE MULTIPLIERS
     F=  3.8203678E+00      P=  4.7097842E+00      G=  3.8203678E+00      RSIGMA=  0.                 H=  0.
  THE CURRENT VALUE OF X IS
   9.4805205E-02       9.4805206E-02       9.4805206E-02       9.4805206F-02
  THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
   4.5624932E-01       9.2482785F-01
  TIME=  152.061 SECONDS
********************************************
        POINT=   9        DOTT=  3.2390600E+00      RHO=  1.0000000F-02      MAGNITUDE=  1.7997389E+00      PHASE= 2
     F=  4.0201711E+00      P=  4.1049827E+00      G=  3.9601711E+00      RSIGMA=  8.4811687F-02     H=  0.
  THE CURRENT VALUE OF X IS
   1.0050428E+00       1.0050428E+00       1.0050428E+00       1.0050428F+00
  THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
   2.0171462E-02       1.0073698F-02
  TIME=  176.878 SECONDS
********************************************
        POINT=  11        DOTT=  2.6602785E-06      RHO=  1.0000000F-02      MAGNITUDE=  1.6310360F-03      PHASE= 2
     F=  4.0201149E+00      P=  4.1049829E+00      G=  3.9601149E+00      RSIGMA=  8.4867987F-02     H=  0.
  THE CURRENT VALUE OF X IS
   1.0050287E+00       1.0050287F+00       1.0050287F+00       1.0050287F+00
  THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
   2.0115744E-02       1.0045678F-02


 2ND  ORDER ESTIMATES
     F=  3.9997910E+00      P=  4.0464510E+00      G=  3.9979968E+00      RSIGMA=  0.                 H=  0.
  THE CURRENT VALUE OF X IS
   9.9994777E-01       9.9994776E-01       9.9994776E-01       9.9994776F-01
  THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
  -2.0810406E-04      -1.0335564E-04


 1ST  ORDER ESTIMATES
     F=  3.9979968E+00      P=  4.0530843E+00      G=  3.9979968E+00      RSIGMA=  0.                 H=  0.
  THE CURRENT VALUE OF X IS
   9.9949921E-01       9.9949920F-01       9.9949920E-01       9.9949920F-01
  THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
  -2.0022856E-03      -1.0008652F-03


LAGRANGE MULTIPLIERS
     F=  3.9979968E+00      P=  4.0530843E+00      G=  3.9979968E+00      RSIGMA=  0.                 H=  0.
  THE CURRENT VALUE OF X IS
   9.9499643E-03       9.9499644E-03       9.9499644E-03       9.9499644F-03
  THE CONSTRAINT VALUES
                        NOT INCLUDING THE NON-NEGATIVITIES
   4.9712295E-01       9.9545294E-01
  TIME=  201.330 SECONDS
********************************************
        POINT=  12        DOTT=  3.2374195E+00      RHO=  1.0000000E-03      MAGNITUDE=  1.7992831E+00      PHASE= 2
     F=  4.0020147E+00      P=  4.0151190E+00      G=  3.9960147E+00      RSIGMA=  1.3104221F-02     H=  0.
  THE CURRENT VALUE OF X IS
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```
 1.0005037E+00       1.0005037E+00       1.0005037E+00       1.0005037E+00
THE CONSTRAINT VALUES
                      NOT INCLUDING THE NON-NEGATIVITIES
 2.0156495E-03       1.0083689E-03
TIME=  234.886 SECONDS

**********************************************
     POINT=  14       DOTT=  2.0009282E-04     RHO=  1.0000000E-03     MAGNITUDE=  1.4145417E-02     PHASE= 2
   F=  4.0020147E+00    P=  4.0151190E+00    G=  3.9960147E+00    RSIGMA=  1.3104221E-02     H=  0.
THE CURRENT VALUE OF X IS
 1.0005037E+00       1.0005037E+00       1.0005037E+00       1.0005037E+00
THE CONSTRAINT VALUES
                      NOT INCLUDING THE NON-NEGATIVITIES
 2.0156495E-03       1.0083689E-03
APPARENTLY ROUNDOFF ERRORS PREVENT A MORE ACCURATE DETERMINATION OF THE MINIMUM OF THIS SUBPROBLEM.


    2ND  ORDER ESTIMATES
   F=  4.0000239E+00    P=  4.0046497E+00    G=  4.0000036E+00    RSIGMA=  0.               H=  0.
THE CURRENT VALUE OF X IS
 1.0000060E+00       1.0000060E+00       1.0000060E+00       1.0000060E+00
THE CONSTRAINT VALUES
                      NOT INCLUDING THE NON-NEGATIVITIES
 2.4754009E-05       1.3068135E-05


    1ST  ORDER ESTIMATES
   F=  4.0000036E+00    P=  4.0051341E+00    G=  4.0000036E+00    RSIGMA=  0.               H=  0.
THE CURRENT VALUE OF X IS
 1.0000009E+00       1.0000009E+00       1.0000009E+00       1.0000009E+00
THE CONSTRAINT VALUES
                      NOT INCLUDING THE NON-NEGATIVITIES
 4.4821830E-06       2.9327844E-06


  LAGRANGE MULTIPLIERS
   F=  4.0000036E+00    P=  4.0051341E+00    G=  4.0000036E+00    RSIGMA=  0.               H=  0.
THE CURRENT VALUE OF X IS
 9.9949656E-04       9.9949657E-04       9.9949657E-04       9.9949657E-04
THE CONSTRAINT VALUES
                      NOT INCLUDING THE NON-NEGATIVITIES
 4.9611801E-01       9.9170060E-01
TIME=  259.399 SECONDS

**********************************************
     POINT=  15       DOTT=  3.2450944E+00     RHO=  1.0000000E-04     MAGNITUDE=  1.8014145E+00     PHASE= 2
   F=  4.0001962E+00    P=  4.0019712E+00    G=  3.9995962E+00    RSIGMA=  1.7749474E-03     H=  0.
THE CURRENT VALUE OF X IS
 1.0000491E+00       1.0000491E+00       1.0000491E+00       1.0000491E+00
THE CONSTRAINT VALUES
                      NOT INCLUDING THE NON-NEGATIVITIES
 1.9711422E-04       9.9742635E-05
TIME=  294.894 SECONDS

**********************************************
     POINT=  17       DOTT=  5.0298191E-04     RHO=  1.0000000E-04     MAGNITUDE=  2.2427258E-02     PHASE= 2
   F=  4.0001900E+00    P=  4.0019714E+00    G=  3.9995900E+00    RSIGMA=  1.7814014E-03     H=  0.
THE CURRENT VALUE OF X IS
 1.0000475E+00       1.0000475E+00       1.0000475E+00       1.0000475E+00
THE CONSTRAINT VALUES
                      NOT INCLUDING THE NON-NEGATIVITIES
 1.9083354E-04       9.6102526E-05
APPARENTLY ROUNDOFF ERRORS PREVENT A MORE ACCURATE DETERMINATION OF THE MINIMUM OF THIS SUBPROBLEM.


    2ND  ORDER ESTIMATES
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```
  F=  3.9999870E+00     P=  4.0004638E+00     G=  3.9999872E+00     RSIGMA=  0.                  H=  0.
 THE CURRENT VALUE OF X IS
  9.9999677E-01       9.9999676E-01       9.9999676E-01       9.9999676E-01
 THE CONSTRAINT VALUES
                      NOT INCLUDING THE NON-NEGATIVITIES
 -1.2096163E-05      -5.3559433E-06


 1ST  ORDER ESTIMATES
  F=  3.9999872E+00     P=  4.0005105E+00     G=  3.9999872E+00     RSIGMA=  0.                  H=  0.
 THE CURRENT VALUE OF X IS
  9.9999681E-01       9.9999680E-01       9.9999680E-01       9.9999680E-01
 THE CONSTRAINT VALUES
                      NOT INCLUDING THE NON-NEGATIVITIES
 -1.1930379E-05      -5.2730559E-06


LAGRANGE MULTIPLIERS
  F=  3.9999872E+00     P=  4.0005105E+00     G=  3.9999872E+00     RSIGMA=  0.                  H=  0.
 THE CURRENT VALUE OF X IS
  9.9995250E-05       9.9995252E-05       9.9995252E-05       9.9995252E-05
 THE CONSTRAINT VALUES
                      NOT INCLUDING THE NON-NEGATIVITIES
  5.2401690E-01       1.0405554E+00
 TIME=  315.442 SECONDS
*********************************************
    POINT=  18       DOTT=  3.7081309E+00     RHO=  1.0000000E-05     MAGNITUDE=  1.7911256E+00     PHASE= 2
  F=  4.0000197E+00     P=  4.0002418E+00     G=  3.9999597E+00     RSIGMA=  2.2209197E-04     H=  0.
 THE CURRENT VALUE OF X IS
  1.0000049E+00       1.0000049E+00       1.0000049E+00       1.0000049E+00
 THE CONSTRAINT VALUES
                      NOT INCLUDING THE NON-NEGATIVITIES
  2.0593770E-05       1.0988132E-05
```

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13. ABSTRACT

This paper documents a computer program to be used in solving nonlinear programming problems with nonlinear programming problems in the constraints. The program, named INSUMT, is used with the standard program, named SUMT, which implements the sequential unconstrained minimization technique for nonlinear programming. SUMT calls INSUMT when it is necessary to solve a nonlinear program in a constraint. The INSUMT program, together with a fairly complete example of its use, is included in the documentation.

Theory and applications of the models which can be solved using this program are treated in two companion papers.

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	ROLE	WT	ROLE	WT	ROLE	WT
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TWO-SIDED OPTIMIZATION						
GAME THEORY						
SUMT						
INSUMT						
COMPUTER PROGRAM						
ALGORITHM						